An Interlude

- Before going on with another approach to information content and considering information flow we deal with a special problem arising in the models presented: the information content of *logical truths*.
- We mainly present the problem here.
- We introduce Hintikka´s solution, but it had no followers and we are not sure whether and how it really works.

Do You Get Information in a Logic Course?

Rational students should engage only in courses where they can learn something. Now, unfortunately, it seems that you can learn nothing in a logic course, if learning something means to acquire some infor-mation, since the information content of logical truths –seen in the light of the approaches considered so far– is: *nothing*!

The Problem in the Syntactic Approach

- Logical truths are not random. They can be completely expected, there are no alternatives to them. Their probability is 1. This means in the syntactic approach, given the definition of information content “I(x)”:

$$I(\alpha)=\log 1/p(\alpha)=\log 1=0.$$

- Logical truth carry no information at all. You learn nothing from them!

The Problem in the Semantic Approach

- Logical truths are true in all possible worlds. The set of the worlds excluded by their truth is ∅, i.e. given the explication “cont( )” of information content in the possible worlds approach: cont(α)=∅.
- Given a probability measure on worlds their information content is the sum of the probability of worlds in ∅, i.e. int(α)=0. Once again you learn nothing from them!

The Problem in Dretske´s Approach

- In Dretske´s externalistic approach to information the problem of logical truth is even more pressing:
  Knowledge is *defined* as the belief that s is F caused by the information that s is F, *given* some natural laws.
- Even natural laws, as given in *all* relevant contexts, "have an informational measure of zero". Logical truth do not cause anyway.

How to Solve the Problem?

- There might be different types of solution:
  a) logical truths carry no information in the sense explained, but are nevertheless of interest because of some other quality.
  b) information is analysed so as to be able to distinguish between some logical truths.
- A kind of syntactic approach can be of type (a), an ontological approach of type (b).

A Syntactic Solution

- Within a semantic approach some *syntactic* features can be given a role:
- The logical truths "p⊃p" and "(∀x)(x=x)" differ syntactically [cf. Carnap´s *intensional isomorphy*], so *differ* in *meaning*! We care about that! (Sameness of meaning being *more* than logical equivalence.)
- The (mental) representations "water" and "H_2O" have different *functional roles* because of their syntax [cf. Dretske]. We mind!

An Ontological Solution

- Even given *intensional isomorphy* in a semantic approach incorporating syntactic features there are logical truths getting the same meaning although being distinct:

  (∀x)Raven(x) ⊃ (∃x)Raven(x) and
  (∀x)Dog(x) ⊃ (∃x)Dog(x) would be example.

- To solve these cases an *ontological* solution is needed which refers to the constituents (resp. the referents of the constituents).

A Fine Grained Approach

- If you don´t care about ontological plenty, you can distinguish

  $(\forall x)Raven(x) \supset (\exists x)Raven(x)$

  $(\forall x)Dog(x) \supset (\exists x)Dog(x)$

  since the one contains the *property* of *being a dog* while the other contains the property *being a raven*.
- Situation semantics is such an approach.

Hintikka´s Approach

- For mainly historical reasons let´s take a look at Hintikka´s approach. He was one of the first to address the problem as a problem of the information content of logical truths.
- He considered the problem in the light of epistemic modal logic, asking for the correctness of two principles of epistemic modal closure.

Normal Modal Logics

- *Normal* modal logics contain a rule of *necessitation:* $\vdash\alpha \rightarrow \vdash\Box\alpha$
  and the *K-Axiom:* $\vdash\Box(\alpha\supset\beta)\supset(\Box\alpha\supset\Box\beta)$,
  i.e. the *derived rule*: $\vdash(\alpha\supset\beta) \rightarrow \vdash(\Box\alpha\supset\Box\beta)$
- Without these there isn´t much of a logic.
- The counterparts in epistemic modal logic
  $\vdash\alpha \rightarrow \vdash K\alpha$ [all logical truth are known]
  $\vdash(\alpha\supset\beta) \rightarrow \vdash(K\alpha\supset K\beta)$ [all consequences are known]
  are considered highly contra-intuitive.

A Little Dialogue

- A: Why didn´t you show up in the exam?
  B: I need not, I know all logical truth.
  A: Wow! How that?
  B: $\vdash\alpha \rightarrow \vdash K\alpha$, you know from modal logic class, don´t you?
  A: Then what about "$\Diamond(\alpha\supset\beta) \wedge \Box\alpha \supset \Diamond\beta$"? From modal logic, right?
  B: Eerr??
- Hintikka tries to avoid these contra-intuitive consequences by distinguishing kinds of information: *surface* vs. *depth* information. But he also restricts the closure principles.

Surface and Depth Information (Outline)

- Hintikka believes there is a sense of information in which logical inference can *add* to our information, i.e. our knowledge.
- His explication relates our problems in recognising a logical truth (i.e. in getting additional information) to the increasing depth of a procedure of checking quantificational consistency.

Surface and Depth Information (Workings)

- *Surface* and *depth* information are defined relative to a nesting of quantifiers.
- Closure (under **K**) does hold only if $\alpha \supset \beta$ is a surface tautology at the depth of α (i.e. at the depth of what is already known).
- Increasing the depth and then detaching (in a conditional) can add to our knowledge. Closure (under **K**) doesn`t apply here!

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Surface and Depth Information (Programme)

- An account of epistemic closure depends on an account of logical depth information (in a first order possible worlds semantics).
- *Depths* informally concerns the finite number of individuals we consider at the same time resp. the number we need to define another individual (given a language L_i).
- We need some *measure* of surface and depth information to compare them.

Surface and Depth Information – Some Questions

- Is this a *psychological* theory? Where from?
- Why are just *quantifiers* the problem? Even though PC is decidable *we* might not be able to discover that some α is a tautology.
- Why not say we don´t know all the consequences of our beliefs, since this surpasses our capacities because of computational complexity (we haven´t enough time and storage) or –in some cases– undecidability?

Surface and Depth Information – Some Details

- A bound variable doesn´t refer to any individual in particular, but we ask whether the definition of the individual concerned refers (by nested quantifiers) to other individuals.
- The *degree* of a formula is obtained as sum of the number of free singular terms [but we leave these out here] and the maximal number of quantifiers whose scopes have a common part in the formula (i.e. its *depth*).

Surface and Depth Information – Some Details(II)

- We can count quantifiers to recognise the depths of the parts of a conclusion.
- Let a *Q-predicate* be the conjunction of all basic predicates or their negation as they apply to some individual. There are as many *types* of individuals as Q-predicates. We consider only types here. Given these predicates we describe a *type* of world by saying which Q-predicates are instantiated.

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Surface and Depth Information – Some Details(III)

- These descriptions of types of worlds are *constituents*. They are consistent.
- If we also allow for basic *relations*, a *Q´-predicate* can be of more than depth 1. If we nest references to other individuals depth increases, as does the depth of constituents.
- These constituents *now* can be inconsistent. So the negation of such a constituent is a logical truth.

Surface and Depth Information – Some Details(IV)

- So at the level of basic predicates we have conjunctions like: $P_1(x) \wedge P_2(x) \wedge \neg P_3(x)$... each giving us a Q-predicate $Q_1(x)$, $Q_2(x)$ ...
- At the level of constituents we have: $(\exists x)Q_1(x) \wedge \neg(\exists x)Q_2(x)$ ...
- Allowing for basic relations in Q_i means that within a Q-predicate „∀“ can occur (i.e. a nesting „∃x∀y“), since referring to another individual is done with definite descriptions „$(\exists x)(\forall y)(\ ...x... \supset x=y)$“.

Surface and Depth Information – Some Details(V)

- Like constituents *logical truths* get assigned a corresponding *depth*. If you formulate these logical truths as conditionals you see which of them are information increasing.
- Checking for consistency is done depth by depth, looking for trivial inconsistency at the subordinate clauses´ depth.

  [This procedure is, of course, not effective – which makes so checking the applicability of closure under **K** non-effective. *Given* that we know α being a logical truth counting its quantificational depth is effective.]

Assessment

- Although Hintikka employs the machinery of the semantic approach the procedure looks cumbersome and non-effective.
- The model may explain why information is gained by consequence, but it doesn`t say *which* information we get.
- The problem of propositional logical truth is not touched.
- That might be reasons to look for another approach (within situation semantics or even AIT).

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AIT on Logical Truth

- Algorithmic Information Theory could be a *syntactic* solution at least to the problem why different logical truths have different information content.
- In AIT one could assume that *one* program capable to generate a string that is a logical truth is it`s *proof*! So logical truth would have definite information content, and different logical truth could have different ones. (Given that we single out *that* program.)

Sources

- We spelled out Carnap´s semantic approach beforehand and turn next to the situational approach; see the references there.
- Papers by Jaakko Hintikka on the problem:
  - several papers in his book
    Logic, Language-Games and Information. Oxford, 1973.
  - his "classical" paper on the subject is:
    "Surface Information and Depth Information", in: Hintikka, J. /Suppes, P. (eds.) *Information and Inference*. Dordrecht, 1970.