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I would like to point out ath all of the natural deduction derivations in this paper were executed using a portable public domain Prolog program I designed and implemented. The program is able to generate either ASCII or LaTeX output. Information on how to get this program and the grammar used for this paper is available from the address below.

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$$\frac{\textit{Jack}}{np\colon \mathbf{j}}\,l \qquad \frac{\textit{likes}}{s\backslash np/np \quad \mathbf{like}}\,l \qquad \frac{\textit{himself}}{s\backslash np\backslash(s\backslash np/np) \quad \lambda V.\lambda x.V(x)(x)}\,d \qquad \frac{\textit{and}}{co\colon \wedge}\,l \qquad \frac{\textit{every student}}{s\backslash np\backslash(s\backslash np/np) \quad \lambda U.\lambda y.\mathbf{every}(\mathbf{student})(\lambda w.U(w)(y))}\,\backslash i^{9}$$

$$\frac{}{s\backslash np\backslash(s\backslash np/np)\colon \lambda T.\lambda z.T(z)(z) \wedge \mathbf{every}(\mathbf{student})(\lambda w.T(w)(z))}\,ce$$

$$\frac{}{s\backslash np\colon \lambda z.\mathbf{like}(z)(z) \wedge \mathbf{every}(\mathbf{student})(\lambda w.\mathbf{like}(w)(z))}\,\backslash e$$

$$\frac{}{s\colon \mathbf{like}(\mathbf{j})(\mathbf{j}) \wedge \mathbf{every}(\mathbf{student})(\lambda w.\mathbf{like}(w)(\mathbf{j}))}\,\backslash e$$

Figure 62: Analysis of *Jack likes himself and every student*

object, as in the following.

(70) John showed Mary herself.

The apparent problem with this example is that the object *Mary* is consumed before *herself* in our system. Morrill [1993] solves this problem with a wrapping-based account of verbs like *show*, an idea originally developed by Bach [1984]. In Morrill's analysis, ditransitive verbs may apply to their more distant object before their nearer one. More specifically, *showed* __ *herself* wraps around *Mary* to generate *showed Mary herself*. Under this approach, a different category is required for the reflexive, but it is a natural analogue of the one we provided above, the only difference being the direction of the bound noun phrase.

(71) *herself* $\Rightarrow s/np{\Uparrow}np\colon \lambda V.\lambda x.V(x)(x)$

To use this category for (70), we would need to employ slash introduction to factor out the subject so that the reflexive could reduce. This category also accounts for other occurrences of reflexives, such as the following.

(72) The picture of himself taken last year still upsets Bill.

Of course, work remains to be done on limiting the possibilities for reflexive binding, as there are well known syntactic limitations on its application. Again, the reader is referred to Morrill [1990, 1992b] for a general account of domains of locality.

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$$
\dfrac{\dfrac{\dfrac{[s\backslash np/np: V]^0 \quad \dfrac{\dfrac{\textit{himself}}{s\backslash np\Uparrow np: \lambda U.\lambda x.U(x)(x)}\,l}{np: y}\,\Uparrow e^3}{s\backslash np: V(y)}\,/e}{s: \lambda x.V(x)(x)}\,\Uparrow e^3}{s\backslash np\backslash(s\backslash np/np): \lambda V.\lambda x.V(x)(x)}\,\backslash i^0
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{[np: x]^4 \quad \dfrac{[s\backslash np/np: V]^0 \quad \dfrac{\dfrac{\textit{every student}}{s\Uparrow np: \mathbf{every}(\mathbf{student})}\,d}{np: y}\,\Uparrow e^5}{s\backslash np: V(y)}\,/e}{s: V(y)(x)}\,\backslash e}{s: \mathbf{every}(\mathbf{student})(\lambda y.V(y)(x))}\,\Uparrow e^5}{s\backslash np: \lambda x.\mathbf{every}(\mathbf{student})(\lambda y.V(y)(x))}\,\backslash i^4}{s\backslash np\backslash(s\backslash np/np): \lambda V.\lambda x.\mathbf{every}(\mathbf{student})(\lambda y.V(y)(x))}\,\backslash i^0
$$

Figure 61: Analysis of *every student* and *himself*

logic remains open.

In our system, though, we are still able to derive an analysis of the following sentence.

(68) Jack likes himself and every student.

The analysis merely involves raising both the reflexive and the quantifier to the category $s\backslash np\backslash(s\backslash np/np)$, which is accomplished by the derivation in Figure 61.[28] The coordination of the two, along with their application, is given in Figure 62.

Our lexical entry for the reflexive accounts for occurrences of reflexives in environments such as the following.

(69) a. John believes he spilled coffee on himself.
b. John treated himself to coffee.
c. John gave a present to himself.
d. John gave himself a present.

What it doesn't account for is cases where a reflexive object binds another

[28] Not coincidentally, this is the category Steedman [1985, 1988] assigns lexically to both object quantifiers and reflexives.

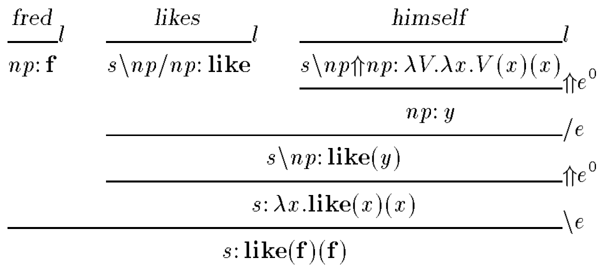

Figure 59: Analysis of *fred likes himself*

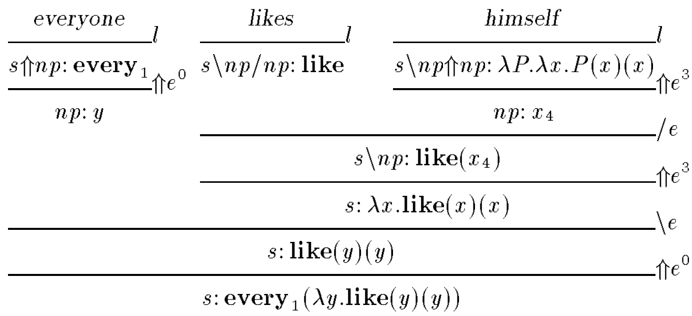

Figure 60: Analysis of *everyone likes himself*

its category, its semantic type is $(\mathsf{Ind} \rightarrow \mathsf{Ind} \rightarrow \mathsf{Prop}) \rightarrow \mathsf{Ind} \rightarrow \mathsf{Prop}$. Thus the variable V in the above is the type of a transitive verb, and its semantics can be seen as taking a transitive verb and reducing its semantics by applying twice to the subject argument x.

The simplest context in which reflexives may apply is as in Figure 59. An example involving a quantified subject is equally straightforward. Such an analysis is provided in Figure 60.

The incompleteness of our logic for the scoping connective can be seen when we try to coordinate a reflexive and a quantifier. We would like to be able to carry out the following derivation.

(67) $s{\Uparrow}np\colon \alpha \Rightarrow s\backslash np{\Uparrow}np\colon \lambda V.\lambda y.\alpha(\lambda x.V(x)(y))$

The reason we want this derivation is because if an expression is of category $s{\Uparrow}np$, then it can act as a noun phrase and reduce in a sentence. But if this is the case, it should also be able to act as a noun phrase and reduce in a verb phrase. This is because we know that such sentential quantifiers can be reduced in the context of verb phrases by the use of slash introduction for the subject, as can be seen in previous derivations, such Figure 28. Our proof theory is sound with respect to our intuitive characterization of $\Uparrow$; the search for a complete

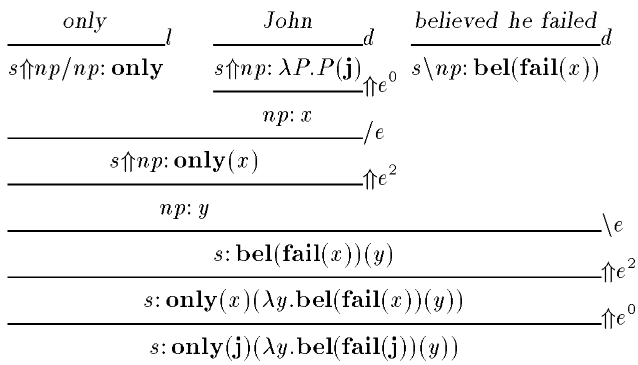

Figure 58: "Strict" Analysis of *only John believed he failed*

comparison class, in this case most likely the other members of the department, much in the same way as comparatives involve implicit comparison classes. Just such an approach is developed by Rooth [1985, 1992], in his general theory of focus-sensitive elements such as *only*.

Possessive pronouns can be analyzed in exactly the same way as non-possessive ones, by taking the pronominal element to be a variable. This gives us the following lexical entry for *its*, for example.

(64) *its* $\Rightarrow np/n\colon \lambda P.\iota(\lambda y.P(y) \wedge \mathbf{poss}(y)(x))$ $\qquad [x \in \mathsf{Var}_{\mathsf{Ind}}]$

Recall that $\mathbf{poss}(y)(x)$ holds if x is the possessor of y. In this case the variable x, for the possessor, is picked up by binding. Thus we get the following analysis for (7)a.

(65) *every Englishman supports his football team* $\Rightarrow$

$$s\colon \mathbf{every}(\mathbf{Englishman})\ (\lambda x.\mathbf{support}(\iota(\lambda y.\mathbf{fb\_team}(y) \wedge \mathbf{poss}(y)(x)))(x))$$

The variable x is simply picked up by the quantifier. Of course, the alternative reading where *his* picks up its reference extra-sententially would involve choosing a different variable for the possessive pronoun.

10.2. *Reflexives*

In this section, we consider the use of reflexive pronouns such as *itself*. Under Moortgat's [1990a, 1991] analysis, reflexives are treated as quantificational elements. In particular, he provides the following lexical entry.

(66) *itself* $\Rightarrow s\backslash np{\Uparrow}np\colon \lambda V.\lambda x.V(x)(x)$

What distinguishes the reflexive here as a quantifier is the fact that it reduces semantically at the verb phrase level, rather than at the sentential level. Given

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textit{only}}{s\Uparrow np/np:\mathbf{only}}l \quad \dfrac{\textit{John}}{np:\mathbf{j}}l}{s\Uparrow np:\mathbf{only}(\mathbf{j})}/e}{np:y}\Uparrow e^0 \quad \dfrac{\dfrac{[np:x]^3 \quad \dfrac{\textit{believed he failed}}{s\backslash np:\mathbf{bel}(\mathbf{fail}(x))}d}{s:\mathbf{bel}(\mathbf{fail}(x))(x)}\backslash e}{s\backslash np:\lambda x.\mathbf{bel}(\mathbf{fail}(x))(x)}\backslash i^3}{s:\mathbf{bel}(\mathbf{fail}(y))(y)}\backslash e}{s:\mathbf{only}(\mathbf{j})(\lambda y.\mathbf{bel}(\mathbf{fail}(y))(y))}\Uparrow e^0$$

Figure 57: "Sloppy" Analysis of *only John believed he failed*

as shown in Figure 57, to derive the so-called *sloppy* reading of a pronoun in a verb phrase such as (7)b, the existence of which was first noted by Geach [1962]. While we do not treat verb-phrase ellipsis, we believe that our approach to anaphora, coupled with a categorial grammar allowing slash introduction, provides a means of analyzing verb phrases in a way that makes them available as "sloppy" antecedents. For instance, consider the following example.

(62) John believes he passed and Bill does too.

Here Bill's belief can either be about Bill's passing or about John's passing. If the verb phrase is analyzed as in Figure 57, then the ellided verb phrase can be identified with it to produce the sloppy reading of (62). No additional mechanism is necessary beyond that which falls out of the appropriate logic for complementation.[27]

The so-called *strict* reading of sentences such as (7)b, where John is the only one who believed John failed, must be derived by one of two means. One method involves standard referential discourse anaphora, as is needed for examples such as (60)a. But an alternative is open to us involving type raising the complement to *only* and then quantifying it over the whole sentence. Such an analysis is given in Figure 58. In general, this approach is acceptable because the complement to *only* does not form a quantification island, as evidenced by the following sentence.

(63) Only the head of every department was invited.

Here we have the universal quantifier taking widest scope. But in addition, we see with this example that there is an additional comparison class involved in such quantification. The reading derived for this sentence, which says for the head of each department, that that head was the only one invited is clearly contradictory. Thus we must be reading the uniqueness with respect to some

[27] For the many other cases of sloppy readings in ellipsis, and related cases of gapping, the approach of Solias [1992] appears promising. It remains to be seen whether or not the flexible categorial approach outlined here solves some of the difficult problems posed for previous "relational" accounts of ellipsis and gapping raised by Dalrymple, Shieber and Pereira's [1991]. Of course, our grammatical approach is compatible with their logical reconstruction approach.

The analysis of simple noun phrases occurring as bound antecedents can be handled in the same way, by simply type-raising the noun phrases to quantifiers and then proceeding as in Figure 56.

We could also provide a categorial analysis more directly in line with Montague's PTQ. Suppose we had a category *pro* $\in$ Cat, which does not associate with a semantic type. We could then use the following sequent rule to mimic Montague's term insertion rule.

(59) $$\frac{\Gamma_0,\ np{:}\,x,\ \Gamma_1,\ np{:}\,x,\ \ldots,\ \Gamma_n,\ np{:}\,x,\ \Delta_0,\ np{:}\,x,\ \Delta_1,\ np{:}\,x,\ \ldots,\ \Delta_m \Rightarrow \phi}{\Gamma_0,\ pro,\ \Gamma_1,\ pro,\ \ldots,\ \Gamma_n,\ s{\Uparrow}np{:}\,\alpha,\ \Delta_0,\ pro,\ \Delta_1,\ pro,\ \ldots,\ \Delta_m \Rightarrow \alpha(\lambda x.\phi)}{\Uparrow}l'$$
$[n, m \geq 0]$

The only difference is that this rule does not allow for the possibility of unbound variables. We will stick to the simpler sequent scheme for quantifier and pronominal entries with variables, but nothing in the present setting hinges on this decision.

There are strict syntactic restrictions on the use of pronouns as bound variables. For instance, consider the following contrast.

(60) a. I like John_1 and he_1 likes me.
b. * I like [every boy]$_1$ and he_1 likes me.

We have already indicated that quantifiers are not allowed to escape from conjuncts to take wide scope over the entire coordinate structure (see [Morrill 1990a, 1990b, 1992a, 1992b] for an account of locality restrictions in categorial grammar consistent with our approach to quantification). This explains why (60)b is ungrammatical. We do not deal with the kind of anaphora required to handle cases such as (60)a. In general, referential occurrences of noun phrases include not only proper names, but also definites such as *the student* and some uses of indefinites such as *a student*.[26]

In addition to the analyses afforded by PTQ, we also derive significant interactions between our categorial system and pronouns. We consider first the sentence (7)b, the analysis of which we adopt from [Carpenter 1989]. The discourse quantifier *only* is able to play many roles, but it is often used with noun phrases to produce an exclusive quantifier. For this case, we can get by with a lexical entry of the following form.

(61) *only* $\Rightarrow s{\Uparrow}np/np{:}\,\mathbf{only}$

We further assume that the interpretation of **only** is such that $\mathbf{only}(x)(P)$ is true if and only if x is interpreted as the only object with the property interpreting P. Pronominal co-reference can then interact with slash introduction,

[26] Following Fodor and Sag [1982], we assume that indefinites are ambiguous between an indefinite quantifier reading and a referential reading. In cases where indefinites provide antecedents, they must be read referentially. For instance, we cannot take *a movie* as both having narrow scope and providing an antecedent for the pronoun in cases such as the following.

Every semanticist read [a paper]$_1$. It_1 was by Montague.

$$\dfrac{\textit{every student}}{s\Uparrow np\colon \mathbf{every}(\mathbf{student})}\,d \qquad \dfrac{\textit{believed}}{s\backslash np/s\colon \mathbf{bel}}\,l \qquad \dfrac{\textit{he}}{np\colon x}\,l \qquad \dfrac{\textit{failed}}{s\backslash np\colon \mathbf{fail}}\,l$$

$$\dfrac{s\Uparrow np\colon \mathbf{every}(\mathbf{student})}{np\colon x}\Uparrow e^0 \qquad \dfrac{np\colon x \quad s\backslash np\colon \mathbf{fail}}{s\colon \mathbf{fail}(x)}\backslash e$$

$$\dfrac{s\backslash np/s\colon \mathbf{bel} \quad s\colon \mathbf{fail}(x)}{s\backslash np\colon \mathbf{bel}(\mathbf{fail}(x))}/e$$

$$\dfrac{np\colon x \quad s\backslash np\colon \mathbf{bel}(\mathbf{fail}(x))}{s\colon \mathbf{bel}(\mathbf{fail}(x))(x)}\backslash e$$

$$\dfrac{s\colon \mathbf{bel}(\mathbf{fail}(x))(x)}{s\colon \mathbf{every}(\mathbf{student})(\lambda x.\mathbf{bel}(\mathbf{fail}(x))(x))}\Uparrow e^0$$

Figure 56: Analysis of *every student believed he failed*

10.1. *Variable Pronouns*

We begin with the pronouns to which we assign a variable semantics, following Montague [1970b]. Our lexical entries for subject and object pronouns take the following general form.[25]

(56) *she* $\Rightarrow np\colon x$ $[x \in \mathsf{Var}_{\mathsf{Ind}}]$

Note that like Montague's analysis, this makes pronouns infinitely ambiguous, as there are countably infinitely many individual variables.

We are able to use these pronominal lexical entries to derive bound occurrences of pronouns, as shown in Figure 56. Here we see the bound-variable reading, which requires the variable introduced by the pronoun to be identical to that introduced by the quantifier. If a different variable were introduced, the resulting reading would be the following.

(57) $\mathbf{every}(\mathbf{student})(\lambda x.\mathbf{bel}(\mathbf{fail}(y))(x))$

For this latter derivation to be useful, we must assume that some discourse process identifies the contents associated with free variables, as in Discourse Representation Theory [Kamp 1984] or File-Change Semantics [Heim 1982]. A similar process might also account for deictic (demonstrative) uses of pronouns, which pick up referents by extra-linguistic means, as in the following examples [Kaplan 1979].

(58) a. Don't go in there. He's awfully upset today.
b. John got a hit. It sailed right out of the park.

In the first case, the occupant of the office might be the antecedent of the pronoun, and in the second case, it is likely the baseball that was hit. But neither referent is available directly from the discourse.

[25] As we mentioned earlier, we are not concerned here with matters of syntactic case. Thus object-position, subject-position and even genitive complement position pronouns such as *hers* are assigned the same categories. As far as agreement goes, we believe the most natural approach is to adopt a referent-based approach such as that developed by Barlow [1988] and by Pollard and Sag [1992, in press]. Such an approach sorts the variables according to the agreement facts of the language in question (gender, number and case in English), and only allows quantifiers to bind variables of a single type.

take either relatively wide or narrow scope. This is accomplished by analyzing the conjuncts in one of two ways, as shown previously in Figure 28, and then carrying out the rest of the reduction by slash elimination. The way in which the conjuncts are analyzed is independent, thus leading to four possible readings for (5)f. Only two of these readings are natural, and they involve parallel analyses of the conjuncts, thus giving the external quantifier either wide or narrow scope with respect to both quantifiers. But we believe such a restriction is pragmatic, rather than syntactic or semantic.

A similar analysis allows us to treat the Partee and Rooth examples as being more ambiguous than originally noted. In general, it is possible in sentences with an intensional verb in which the object is the coordination of two quantifiers to allow one of the conjuncts to be read de dicto, and the other de re. For instance, consider the following sentences.

(55) a. The princess sought [a sword]$_1$ and [a knight to wield it$_1$].
b. The tourist sought a shop he had seen the day before or another one with similar merchandise.
c. The professor was looking for a pen she misplaced and something on which to write.

In the first example, the princess could have a particular sword in mind, thus taking it to be de re, while the desire for the knight to wield it might be de dicto. These mixed de dicto/de re examples are generated by type raising the objects, one in a de dicto fashion and one in a de re fashion before coordinating them. In the second example, a particular shop from the day before could be de re, even if there were many such shops, while the new one would most likely be read de dicto. As we will see in the next section, for the coreference to work with the pronoun in the first example, and with the adjective *similar* in the second, the first conjunct in both cases must be read de re. Again, we believe the reason such usages are not more common is a pragmatic tendency to read coordinate structures in a parallel fashion.

10. Quantification and Pronominal Dependency

In this section, we take up the issue of the reference of third-person pronominals in cases in which they are bound intra-sententially by quantifiers. For standard pronouns, such as subject-position *he*, object-position *his*, and determiner *his*, we follow Montague in providing a variable for the semantics of the bound element. For reflexives, such as *herself*, we follow Moortgat [1990a, 1991] in employing a binding operator, along the lines of our approach to quantifiers. We do not consider pronouns bound by discourse conventions either within or across clauses. We also do not discuss indexical pronouns such as *me*, *you* or *now*; our analysis is compatible with those in which indexicals pick up their references by contextual means [Bar-Hillel 1954, Montague 1970b].

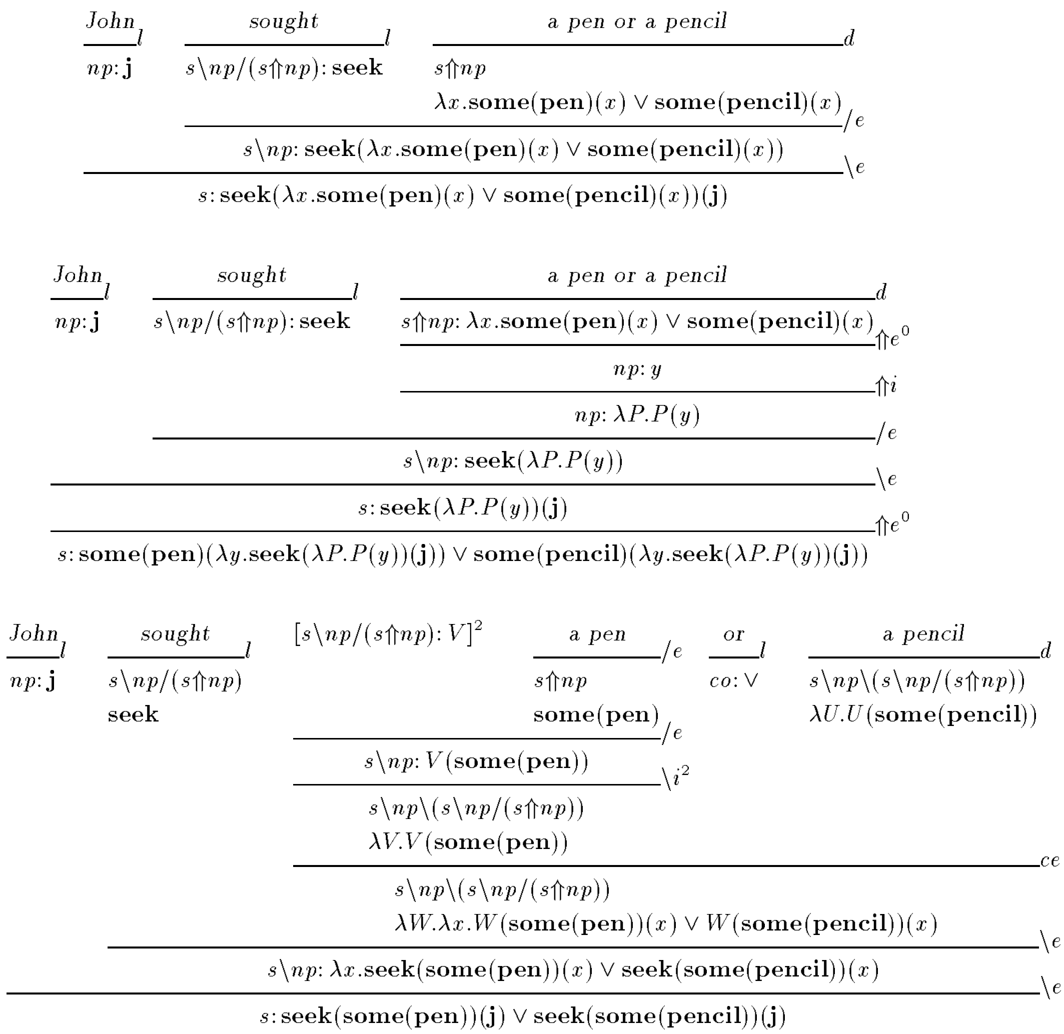

Figure 55: Analysis of *John sought a pen or a pencil*

verbs take coordinated objects, as evidenced by examples such as (5)e. We provide the three analyses in Figure 55. Continuing with the examples in (5), we have a case in (5)f in which two "non-constituents" containing a quantifier are coordinated. The coordinated element can be analyzed using slash introduction as seen previously in Figure 33. A similar analysis for *every student disliked* along with the obvious application of coordination and slash elimination yields the unique reading of (5)f.

The final case we consider is (5)g, where there is a quantifier embedded in a conjunct that interacts with a quantifier outside of the conjunct, in that it can

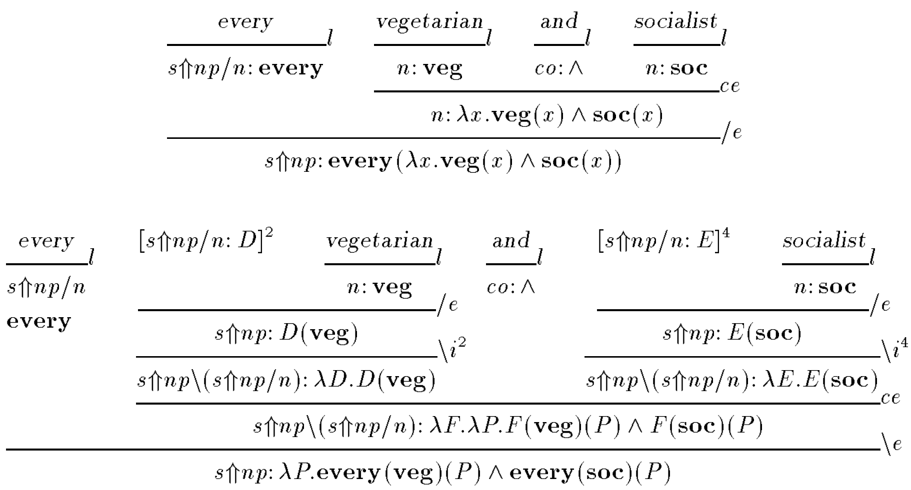

Figure 53: Analysis of *every vegetarian and socialist*

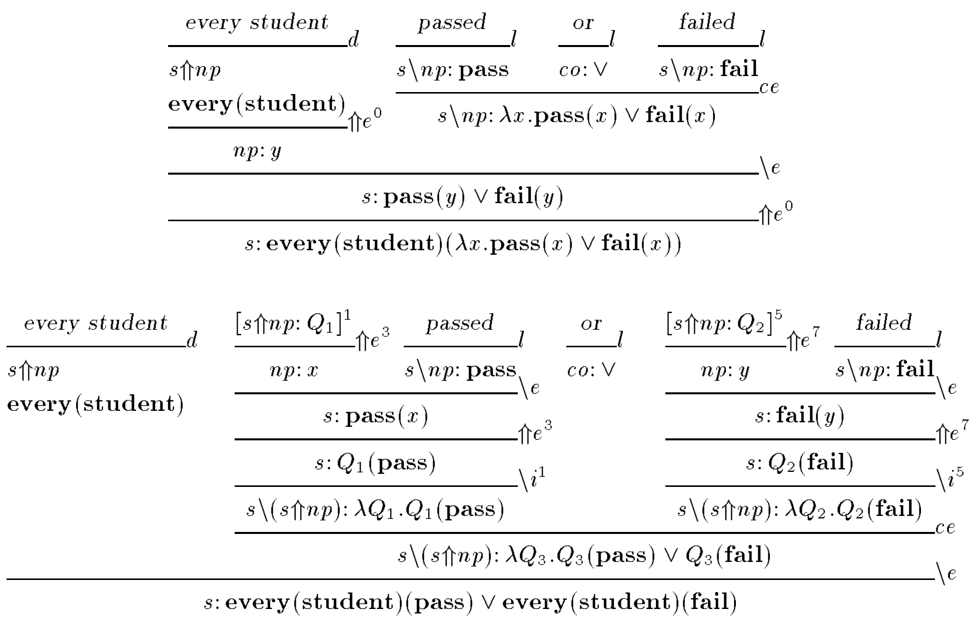

Figure 54: Analysis of *every student passed or failed*

in (5)d. The two analyses of this sentence are provided in Figure 54. A similar form of analysis, involving hypothetical reasoning for type raising, accounts for the ambiguities noted by Partee and Rooth [1983], which arise when intensional

$$\frac{\dfrac{\dfrac{\textit{John}}{np\colon \mathbf{j}}\,l}{np\colon \lambda P.P(\mathbf{j})}\Uparrow i \qquad \dfrac{\textit{and}}{co\colon \wedge}\,l \qquad \dfrac{\textit{every prof}}{s\Uparrow np\colon \mathbf{every}(\mathbf{prof})}\,d}{s\Uparrow np\colon \lambda R.R(\mathbf{j}) \wedge \mathbf{every}(\mathbf{prof})(R)}\,ce$$

Figure 51: Analysis of *John and every prof*

$$\dfrac{\textit{a student}}{s\Uparrow np\colon \mathbf{some}(\mathbf{student})}\,d \qquad \dfrac{\textit{likes}}{s\backslash np/np\colon \mathbf{like}}\,l \qquad \dfrac{\textit{John and every prof}}{s\Uparrow np\colon \lambda P.P(\mathbf{j}) \wedge \mathbf{every}(\mathbf{prof})(P)}\,d$$

$$\dfrac{s\Uparrow np\colon \mathbf{some}(\mathbf{student})}{np\colon x}\Uparrow e^0 \qquad \dfrac{s\Uparrow np\colon \lambda P.P(\mathbf{j}) \wedge \mathbf{every}(\mathbf{prof})(P)}{np\colon y}\Uparrow e^3$$

$$\dfrac{s\backslash np/np\colon \mathbf{like} \quad np\colon y}{s\backslash np\colon \mathbf{like}(y)}\,/e$$

$$\dfrac{np\colon x \quad s\backslash np\colon \mathbf{like}(y)}{s\colon \mathbf{like}(y)(x)}\,\backslash e$$

$$\dfrac{s\colon \mathbf{like}(y)(x)}{s\colon \mathbf{like}(\mathbf{j})(x) \wedge \mathbf{every}(\mathbf{prof})(\lambda y.\mathbf{like}(y)(x))}\Uparrow e^3$$

$$\dfrac{s\colon \mathbf{like}(\mathbf{j})(x) \wedge \mathbf{every}(\mathbf{prof})(\lambda y.\mathbf{like}(y)(x))}{s\colon \mathbf{some}(\mathbf{student})(\lambda x.\mathbf{like}(\mathbf{j})(x) \wedge \mathbf{every}(\mathbf{prof})(\lambda y.\mathbf{like}(y)(x)))}\Uparrow e^0$$

$$\dfrac{\textit{a student}}{s\Uparrow np\colon \mathbf{some}(\mathbf{student})}\,d \qquad \dfrac{\textit{likes}}{s\backslash np/np\colon \mathbf{like}}\,l \qquad \dfrac{\textit{John and every prof}}{s\Uparrow np\colon \lambda P.P(\mathbf{j}) \wedge \mathbf{every}(\mathbf{prof})(P)}\,d$$

$$\dfrac{s\Uparrow np\colon \mathbf{some}(\mathbf{student})}{np\colon x}\Uparrow e^2 \qquad \dfrac{s\Uparrow np\colon \lambda P.P(\mathbf{j}) \wedge \mathbf{every}(\mathbf{prof})(P)}{np\colon y}\Uparrow e^0$$

$$\dfrac{s\backslash np/np\colon \mathbf{like} \quad np\colon y}{s\backslash np\colon \mathbf{like}(y)}\,/e$$

$$\dfrac{np\colon x \quad s\backslash np\colon \mathbf{like}(y)}{s\colon \mathbf{like}(y)(x)}\,\backslash e$$

$$\dfrac{s\colon \mathbf{like}(y)(x)}{s\colon \mathbf{some}(\mathbf{student})(\mathbf{like}(y))}\Uparrow e^2$$

$$\dfrac{s\colon \mathbf{some}(\mathbf{student})(\mathbf{like}(y))}{s\colon \mathbf{some}(\mathbf{student})(\mathbf{like}(\mathbf{j})) \wedge \mathbf{every}(\mathbf{prof})(\lambda y.\mathbf{some}(\mathbf{student})(\mathbf{like}(y)))}\Uparrow e^0$$

Figure 52: Analysis of *a student likes John and every prof*

in Figure 51. The coordination of a pair of quantifiers acts just like other quantifiers in their ability to scope. This allows us to derive scope ambiguities as shown in Figure 52.

The interaction between coordination and quantification becomes more interesting when the scope or restriction of a quantifier is coordinated. The correct results follow from our logical approach without the addition of further mechanisms. For instance, consider the two possible analyses of the subject of (5)c, given in Figure 53. The first of these analyses quantifies over individuals who are both socialists and vegetarians. The second provides a quantifier which universally quantifiers over vegetarians and over socialists independently.

A similar analysis accounts for examples where the scope is coordinated, as

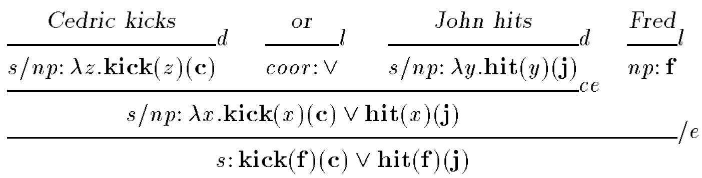

Figure 48: Analysis of *John hits or Cedric kicks*

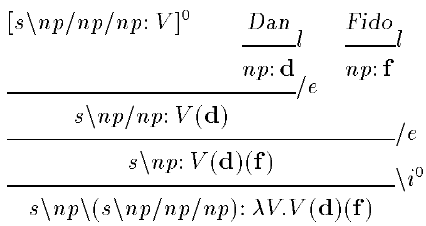

Figure 49: Analysis of *Dan Fido*

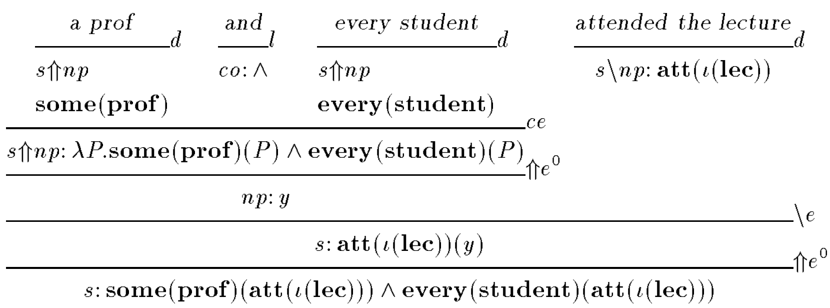

Figure 50: Analysis of *a prof and every student attended the lecture*

Here we simply use our standard hypothetical reasoning, as shown in Figure 49. Such an analysis can be used to produce the following derivation.

(54) *Max sold [Tim Fido] and [Joe Felix]* ⇒
s: **sell(t)(fido)(m)** ∧ **sell(j)(felix)(m)**

We now turn to the interaction of quantification and coordination. We begin by considering the coordination of quantifiers, one of the motivating examples for PTQ [Montague 1970b]. For instance, the analysis of an example like that in (5)a is given in Figure 50. Here we see that quantifiers can be coordinated because they are of boolean type. We can also coordinate noun phrases and quantifiers by simply type raising the noun phrases. Such an example is given

Coordination Left

$$\frac{}{A\colon \phi \quad co\colon \alpha \quad A\colon \psi \Rightarrow A\colon Coor(\alpha)(\phi)(\psi)}\,Col$$

Figure 45: Coordination Sequent Scheme

Coordination Elimination

$$\frac{\begin{matrix}\vdots & \vdots & \vdots \\ A\colon \phi & co\colon \alpha & A\colon \psi\end{matrix}}{A\colon Coor(\alpha)(\phi)(\psi)}\,ce$$

Figure 46: Coordination Sequent Scheme

$$\dfrac{\dfrac{\dfrac{\dfrac{in}{n\backslash n/np\colon \mathbf{in}}\,l \quad \dfrac{or}{co\colon \vee}\,l \quad \dfrac{on}{n\backslash n/np\colon \mathbf{on}}\,l}{n\backslash n/np\colon \lambda x.\lambda P.\lambda y.(\mathbf{in}(x)(P)(y)) \vee (\mathbf{on}(x)(P)(y))}\,ce \quad \dfrac{\dfrac{the}{np/n\colon \iota}\,l \quad \dfrac{box}{n\colon \mathbf{box}}\,l}{np\colon \iota(\mathbf{box})}\,/e}{}}{n\backslash n\colon \lambda P.\lambda y.\mathbf{in}(\iota(\mathbf{box}))(P)(y) \vee \mathbf{on}(\iota(\mathbf{box}))(P)(y)}\,/e$$

Figure 47: Analysis of *in or on the box*

this analysis would combine with a determiner and a noun phrase by slash elimination to produce the following analysis:

(52) *the dog in or on the box* $\Rightarrow$
$np\colon \iota(\lambda x.\mathbf{in}(\iota(\mathbf{box}))(\mathbf{dog}) \vee \mathbf{on}(\iota(\mathbf{box}))(\mathbf{dog}))$

The analysis of coordination in categorial grammar is best known for its ability to derive the correct results for so-called "non-constituent" coordination. Steedman's [1985] approach combined type raising and composition, and was later extended by Dowty [1988]. Lambek's categorial grammar, with its hypothetical reasoning mechanism, generalizes the notion of type raising and composition to allow fully flexible coordination. For instance, consider the analyses in Figure 48. For a similar example, recall the derivation in Figure 7, from which we are able to analyze the following coordinate sentence:

(53) *[Bill hit] and [Fred believes Cedric kicked] Mark* $\Rightarrow$
$s\colon \mathbf{hit}(\mathbf{m})(\mathbf{b}) \wedge \mathbf{believe}(\mathbf{kick}(\mathbf{m})(\mathbf{c}))(\mathbf{f})$

A more interesting example involves the propositional analysis of a sequence of two noun phrases, as first analyzed in Steedman's framework by Dowty [1988].

types is defined according to the following recursive scheme.

(49) a. $Coor(\alpha)(\phi)(\psi) = \alpha(\phi)(\psi)$ if ϕ and ψ are of type Prop

b. $Coor(\alpha)(\phi)(\psi) = \lambda x.Coor(\alpha)(\phi(x))(\psi(x))$ if ϕ and ψ are of type $\sigma \rightarrow \tau$, τ a propositional type, and x a fresh variable of type σ

More schematically, we have the following.

(50) $$Coor(\alpha)(\phi)(\psi) = \lambda x_1.\lambda x_2.\ldots.\lambda x_n.\alpha(\phi(x_1)(x_2)\cdots(x_n))(\psi(x_1)(x_2)\cdots(x_n))$$

if ϕ and ψ are of type $\sigma_1 \rightarrow \sigma_2 \rightarrow \cdots \rightarrow \sigma_n \rightarrow \mathsf{Prop}$

This exhausts the semantics of coordination as a propositional operator. The discourse functions of coordination are by no means trivial, but fall beyond the scope of this paper.

Syntactically, we allow arbitrary categories with propositional types to be coordinated. Two related approaches to such coordination have been prevalent in categorial grammar, one schematic and one lexical. The schematic approach employs an inference scheme to carry out coordination, whereas the lexical approach uses polymorphic lexical entries for coordinators. These methods are identical in their semantic predictions.[23] The lexical approach, developed by Morrill [1990b, 1992a], provides a polymorphic lexical entry for coordinators, such as the following:[24]

(51) *and* $\Rightarrow A\backslash A/A$: $Coor(\wedge)$ [$Typ(A)$ is propositional]

Of course, this polymorphic lexical scheme has infinitely many instantiations, one for each category with a propositional type.

The schematic approach is based on adding a new deductive scheme. We introduce a new category $co \in \mathsf{Cat}$ directly, as it does not need to interact with other categories, and thus does not need to be a member of BasCat. We assume co is of type $\mathsf{Prop} \rightarrow \mathsf{Prop} \rightarrow \mathsf{Prop}$. The coordination schemes match the effects of the polymorphic lexical entry given above. The sequent version of coordination is provided in Figure 45, and its natural deduction counterpart in Figure 46. An example of a simple coordination involving nominal prepositions is given in Figure 47. For simplicity, the prepositional arguments are lowered to noun phrases and their semantic contents taken to be constants. Of course,

[23] The syntactic distinctions expressible using the two approaches are beyond the scope of this paper. For instance, we must block the coordination of two coordinators or coordinators which have taken an argument, as evidenced by **ran [[and] but not [or]] jumped.* See [Morrill 1992a] for further details.

[24] Not every category with a propositional type naturally occurs in coordinate structures, though the boundary between he coordinable categories and the non-coordinable ones is not well established. We will simply show how all categories with propositional types can be coordinated, and leave aside potential syntactic restrictions. Because they do not introduce additional semantic effects, we also pass over coordination of more than two expressions, as in *ran, biked and swam*, and wrapping coordinators, as in *either ran or jumped*. A general approach to particles and wrapping is provided by Morrill and Solias [1992].

derivable for it from our grammar.

(48) a. What every student wrote upset a professor.

b. $\mathbf{every}(\mathbf{student})$
$(\lambda x.\mathbf{some}(\mathbf{prof})$
$(\lambda y.\mathbf{upset}(y)(\iota(\lambda z.\mathbf{write}(z)(x)))))$

c. $\mathbf{some}(\mathbf{prof})$
$(\lambda y.\mathbf{every}(\mathbf{student})$
$(\lambda x.\mathbf{upset}(y)(\iota(\lambda z.\mathbf{write}(z)(x)))))$

d. $\mathbf{some}(\mathbf{prof})$
$(\lambda y.\mathbf{upset}(y)(\iota(\lambda z.\mathbf{every}(\mathbf{student})$
$(\lambda x.\mathbf{write}(z)(x)))))$

The first of these readings is where a possibly different professor was upset by the possibly different writing of each student. The second is where there was a single professor who was upset by the possibly different writings of each student. The last case, where the quantifier remains embedded, must be derived by a method analogous to that used in Figure 33. It represents the case in which there was a particular professor who was upset by what the students wrote, with the further requirement that each student wrote the same thing. Thus, for the case of free relatives, island constraints do not prevent embedded quantifiers from escaping to take sentential scope. In fact, the first and second readings given above are of this variety and are quite plausible for this sentence.

9. Quantification and Coordination

Categorial grammar has been particularly successful in its applications to coordination [Steedman 1985, 1991; Dowty 1988; Solias 1992]. In this section, we demonstrate how these results can be extended to coordinate structures involving quantifiers in contexts such as those in (5). Our semantic treatment of coordination preserves the fundamental insights of Montague [1970b], but our categorial approach to syntax allows us to extend such analyses far beyond what he achieved with his syntactic framework.

Semantically, our approach to coordination is a straightforward generalized propositional one along the lines of [Gazdar 1980], which allows arbitrary propositional types to be coordinated. The set of *propositional types* is defined as the least set such that: Prop is a propositional type, and $\sigma \rightarrow \tau$ is a propositional type if τ is a propositional type. In other words, a propositional type is either a proposition or a functional type whose range type is propositional. In general, a propositional type is of the form $\sigma_1 \rightarrow \sigma_2 \rightarrow \cdots \rightarrow \sigma_n \rightarrow \mathsf{Prop}$ for some $n \geq 0$. Coordinators are propositional functions taking two propositions to produce a propositional result, and are thus of type $\mathsf{Prop} \rightarrow \mathsf{Prop} \rightarrow \mathsf{Prop}$. In the simplest case, terms of coordinator type can be applied to pairs of propositions to produce a proposition. But in general, coordinators can be applied to arbitrary terms of matching propositional type by distributing the coordinator. The result of applying a coordinator α to terms ϕ and ψ of matching propositional

every student	*'s*	*approval*
$\overline{s{\Uparrow}np\colon \mathbf{every}(\mathbf{student})}^{\,d}$	$\overline{np/n\backslash(s{\Uparrow}np)\colon \lambda x_5.\lambda x_3.\iota(\lambda x_2.x_3(x_2)\wedge x_5(\mathbf{poss}(x_2)))}^{\,l}$	$\overline{n\colon \mathbf{approval}}^{\,l}$
$\overline{np\colon x_1}^{\,{\Uparrow}e^0}$		
$\overline{np\colon \lambda x_4.x_4(x_1)}^{\,{\Uparrow}i}$		

$$\frac{np\colon \lambda x_4.x_4(x_1) \quad np/n\backslash(s{\Uparrow}np)\colon \lambda x_5.\lambda x_3.\iota(\lambda x_2.x_3(x_2)\wedge x_5(\mathbf{poss}(x_2)))}{np/n\colon \lambda x_3.\iota(\lambda x_2.x_3(x_2)\wedge \mathbf{poss}(x_2)(x_1))}\backslash e$$

$$\frac{np/n\colon \lambda x_3.\iota(\lambda x_2.x_3(x_2)\wedge \mathbf{poss}(x_2)(x_1)) \quad n\colon \mathbf{approval}}{np\colon \iota(\lambda x_2.\mathbf{approval}(x_2)\wedge \mathbf{poss}(x_2)(x_1))}/e$$

$$\frac{np\colon \iota(\lambda x_2.\mathbf{approval}(x_2)\wedge \mathbf{poss}(x_2)(x_1))}{s{\Uparrow}np\colon \lambda P.\mathbf{every}(\mathbf{student})(\lambda z.P(\iota(\lambda y.\mathbf{approval}(y)\wedge \mathbf{poss}(y)(x))))}*$$

Figure 44: Blocked Analysis of *every student's approval*

We have no problem generating the analyses in which the universal quantifier takes wide scope. The problem is that we also need to generate a de dicto reading of the embedded quantifier. For instance, the dean might not even know all of the students in (45)a. In (45)b, the student might not even have a journal in mind. To derive these readings, we need the following analysis of the object of (45).

(46) *every student's approval* ⇒
$$s{\Uparrow}np\colon \lambda P.\mathbf{every}(\mathbf{student})(\lambda z.P(\iota(\lambda y.\mathbf{approval}(y)\wedge \mathbf{poss}(y)(x))))$$

But such an analysis is not derivable using our proof theory, as can be seen from Figure 44. The example in (45)c shows that we can not cure this problem with a more ambitious lexical entry for the possessive marker. But the fault is not due to our general logical approach, but is merely caused by the incomplete proof theory in which we have chosen to couch it. According to our intuitive explanation of the scoping constructor, $\Uparrow$, the analysis in Figure 44 should go through, because the expression is such that it can act as an *np* and reduce in a sentence with the semantic effect shown. A complete logic for quantifiers remains an open problem, though the approach of Morrill and Solias [1992, Morrill 1992a] is a step in the right direction. A further example of our logic's incompleteness arises in the section below on reflexives.

Moving on to the free relative, we provide the following lexical entry, similar in spirit to the base generated analysis of Bresnan and Grimshaw [1978].[22]

(47) *what* ⇒ $np/(s{\uparrow}np)\colon \iota$

Free relatives have interesting interactions with quantifiers, of the kind illustrated in (2)d. A related example is given below, along with the analyses

[22] Rather than the universal interpretation assigned by Cooper [1983], we take the universal force of free relatives under some circumstances to be an instance of genericity. In general, we assume a generic reading to be possible for any definite or indefinite noun phrase.

We also do not consider the interesting case of *whatever*, although it appears to have some kind of quantificational force, because the semantic nature of the quantification involved is rather unclear [Cooper 1983; Bresnan and Grimshaw 1978].

$$\frac{\textit{the}}{np/n\colon \iota}\,l \quad \frac{\textit{receptionist}}{n\colon \mathbf{rec}}\,l \quad \frac{\textit{in}}{n\backslash n/(s\Uparrow np)\quad \lambda Q.\lambda P.\lambda y.P(y)\wedge Q(\lambda x.\mathbf{in}(x)(y))}\,l \quad \frac{\textit{every office}}{s\Uparrow np\quad \mathbf{every}(\mathbf{off})}\,d \quad \frac{\textit{is underpaid}}{s\backslash np\quad \mathbf{underpaid}}\,l$$

$$\frac{}{n\backslash n\colon \lambda P.\lambda y.P(y)\wedge \mathbf{every}(\mathbf{off})(\lambda x.\mathbf{in}(x)(y))}\,/e$$

$$\frac{}{n\colon \lambda y.\mathbf{rec}(y)\wedge \mathbf{every}(\mathbf{off})(\lambda x.\mathbf{in}(x)(y))}\,\backslash e$$

$$\frac{}{np\colon \iota(\lambda y.\mathbf{rec}(y)\wedge \mathbf{every}(\mathbf{off})(\lambda x.\mathbf{in}(x)(y)))}\,/e$$

$$\frac{}{s\colon \mathbf{underpaid}(\iota(\lambda y.\mathbf{rec}(y)\wedge \mathbf{every}(\mathbf{off})(\lambda x.\mathbf{in}(x)(y))))}\,\backslash e$$

$$\frac{\textit{the}}{np/n\colon \iota}\,l \quad \frac{\textit{receptionist}}{n\colon \mathbf{rec}}\,l \quad \frac{\textit{in}}{n\backslash n/(s\Uparrow np)\quad \lambda Q.\lambda P.\lambda y.P(y)\wedge Q(\lambda z.\mathbf{in}(z)(y))}\,l \quad \frac{\textit{every office}}{s\Uparrow np\quad \mathbf{every}(\mathbf{off})}\,/e \quad \frac{\textit{is underpaid}}{s\backslash np\quad \mathbf{underpaid}}\,l$$

$$\frac{s\Uparrow np\quad \mathbf{every}(\mathbf{off})}{np\colon x}\,\Uparrow e^0 \qquad \frac{np\colon x}{np\colon \lambda R.R(x)}\,\Uparrow i$$

$$\frac{}{n\backslash n\colon \lambda P.\lambda y.P(y)\wedge \mathbf{in}(x)(y)}\,/e$$

$$\frac{}{n\colon \lambda y.\mathbf{rec}(y)\wedge \mathbf{in}(x)(y)}\,\backslash e$$

$$\frac{}{np\colon \iota(\lambda y.\mathbf{rec}(y)\wedge \mathbf{in}(x)(y))}\,/e$$

$$\frac{}{s\colon \mathbf{underpaid}(\iota(\lambda y.\mathbf{rec}(y)\wedge \mathbf{in}(x)(y)))}\,\backslash e$$

$$\frac{}{s\colon \mathbf{every}(\mathbf{off})(\lambda x.\mathbf{underpaid}(\iota(\lambda y.\mathbf{rec}(y)\wedge \mathbf{in}(x)(y))))}\,\Uparrow e^0$$

Figure 42: Analysis of *the receptionist in every office is underpaid*

$$\frac{\textit{every student}}{s\Uparrow np\colon \mathbf{every}(\mathbf{student})}\,d \quad \frac{\textit{'s}}{np/n\backslash(s\Uparrow np)\quad \lambda Q.\lambda P.\iota(\lambda x.P(x)\wedge Q(\mathbf{poss}(x)))}\,l \quad \frac{\textit{teacher}}{n\colon \mathbf{teacher}}\,l$$

$$\frac{}{np/n\colon \lambda P.\iota(\lambda x.P(x)\wedge \mathbf{every}(\mathbf{student})(\mathbf{poss}(x)))}\,\backslash e$$

$$\frac{}{np\colon \iota(\lambda x.\mathbf{teacher}(x)\wedge \mathbf{every}(\mathbf{student})(\mathbf{poss}(x)))}\,/e$$

Figure 43: Analysis of *every student's teacher*

The following examples pose a problem for our incomplete logic of quantification.

(45) a. The dean sought every student's approval.
b. The student sought a journal's acceptance.
c. John was seeking a conversation with every speaker at the conference.

$$
\dfrac{\dfrac{\dfrac{\textit{the}}{np/n\colon \iota}\,l \qquad \dfrac{\textit{student}}{n\colon \mathbf{student}}\,l}{np\colon \iota(\mathbf{student})}\,/e \qquad \dfrac{\textit{studied}}{s\backslash np\colon \mathbf{study}}\,l}{s\colon \mathbf{study}(\iota(\mathbf{student}))}\,\backslash e
$$

Figure 41: Analysis of *the student studied*

determiners, as in *every professor*, which is often not taken to refer to every professor in the world, but merely to those in the appropriate context. We will not propose a treatment for such contextualization, as it is a pragmatic rather than semantic phenomena.

Our primary concern here is the interaction of definite descriptions with quantifiers, which our theory correctly characterizes. For instance, consider the two derivation of (2)f given in Figure 42. In the first of these derivations, we have used a simple noun phrase complement category for the preposition. This category is derivable from the more complex category for the preposition given in the second example by a combination of slash introduction and type raising. These derivations show that under our approach, definites can take the appropriate scopes with respect to quantifiers.

Our treatment of possessives follows that of definites. The lexical entries used to generate the example in (2)d is as follows.

(43) $\text{'s} \Rightarrow np/n\backslash(s{\Uparrow}np)\colon \lambda Q.\lambda P.\iota(\lambda y.P(y) \wedge Q(\lambda x.\mathbf{poss}(y)(x)))$

Note that the constant **poss** introduced for the possession portion of the possessive is shown in non-reduced format; applying η-reduction yields $Q(\mathbf{poss}(y))$ as a subterm in its lexical entry. The exact meaning of **poss** does not concern us here, nor do the pragmatic restrictions on when possessives can grammatically occur.[21]

The narrow scope analysis of a quantifier is derived by application and is shown in Figure 43. The wide scope analysis of quantifiers in possessive position is analogous to other wide-scope occurrences: quantifier elimination is used to introduce a noun phrase, which is then type raised and the function applied to it, and the quantifier is allowed to reduce at an appropriate higher level position. This allows for the ambiguity in examples such as the following.

(44) A teaching assistant marked every student's exam.

Here there is an ambiguity as to whether the same teaching assistant marked each of the exams, as well as the ambiguity as to whether there was one exam for the students together, or whether each student had a separate exam.

[21] Barker [1991] provides an in-depth analysis of both the proper semantics and use of possessives that is compatible with our approach. He notes, for instance, that we can say things like *the chair's leg* but not usually *the leg's chair*. Of course, this is clearly not a case of possession except in the broadest sense.

$$
\begin{array}{l}
\dfrac{\textit{some journal}}{s\Uparrow np\colon \mathbf{some}(\mathbf{jour})}d \quad \dfrac{\textit{accepted}}{s\backslash np/np\colon \mathbf{acc}}l \quad \dfrac{[s\Uparrow np\colon Q]^0}{np\colon y}\Uparrow e^5 \\
\dfrac{s\Uparrow np\colon \mathbf{some}(\mathbf{jour})}{np\colon x}\Uparrow e^4 \quad \dfrac{s\backslash np/np\colon \mathbf{acc} \quad np\colon y}{s\backslash np\colon \mathbf{acc}(y)}/e \\
\dfrac{np\colon x \quad s\backslash np\colon \mathbf{acc}(y)}{s\colon \mathbf{acc}(y)(x)}\backslash e \\
\dfrac{s\colon \mathbf{acc}(y)(x)}{s\colon Q(\lambda y.\mathbf{acc}(y)(x))}\Uparrow e^5 \\
\dfrac{s\colon Q(\lambda y.\mathbf{acc}(y)(x))}{s\colon \mathbf{some}(\mathbf{jour})(\lambda x.Q(\lambda y.\mathbf{acc}(y)(x)))}\Uparrow e^4 \\
\dfrac{s\colon \mathbf{some}(\mathbf{jour})(\lambda x.Q(\lambda y.\mathbf{acc}(y)(x)))}{s\uparrow(s\Uparrow np)\colon \lambda Q.\mathbf{some}(\mathbf{jour})(\lambda x.Q(\lambda y.\mathbf{acc}(y)(x)))}\uparrow i^0
\end{array}
$$

$$
\begin{array}{l}
\dfrac{\textit{some journal}}{s\Uparrow np\colon \mathbf{some}(\mathbf{jour})}d \quad \dfrac{\textit{accepted}}{s\backslash np/np\colon \mathbf{acc}}l \quad \dfrac{[s\Uparrow np\colon Q]^0}{np\colon y}\Uparrow e^3 \\
\dfrac{s\Uparrow np\colon \mathbf{some}(\mathbf{jour})}{np\colon x}\Uparrow e^4 \quad \dfrac{s\backslash np/np\colon \mathbf{acc} \quad np\colon y}{s\backslash np\colon \mathbf{acc}(y)}/e \\
\dfrac{np\colon x \quad s\backslash np\colon \mathbf{acc}(y)}{s\colon \mathbf{acc}(y)(x)}\backslash e \\
\dfrac{s\colon \mathbf{acc}(y)(x)}{s\colon \mathbf{some}(\mathbf{jour})(\mathbf{acc}(y))}\Uparrow e^4 \\
\dfrac{s\colon \mathbf{some}(\mathbf{jour})(\mathbf{acc}(y))}{s\colon Q(\lambda y.\mathbf{some}(\mathbf{jour})(\mathbf{acc}(y)))}\Uparrow e^3 \\
\dfrac{s\colon Q(\lambda y.\mathbf{some}(\mathbf{jour})(\mathbf{acc}(y)))}{s\uparrow(s\Uparrow np)\colon \lambda Q.Q(\lambda y.\mathbf{some}(\mathbf{jour})(\mathbf{acc}(y)))}\uparrow i^0
\end{array}
$$

Figure 40: Analysis of *some journal accepted*

This is for the same reason that such analyses can not be generated in the standard cases of relative clauses — derivations must be nested.

8. Definites and Quantifiers

We assume a referential treatment of the definite determiner *the*, in which it behaves as a function from properties to the unique individuals satisfying them. The appropriate lexical entry for this treatment is:

(42) *the* $\Rightarrow np/n\colon \iota$

As usual, ι is the description operator of type $(\mathsf{Ind}\rightarrow\mathsf{Prop})\rightarrow\mathsf{Prop}$, with the restriction that $\iota(P) = x$ if x is the unique individual such that $P(x)$ is true (see [Andrews 1986], for example). We will not worry about the converse condition when there is no x or more than one x such that $P(x)$ is true, as we assume this situation corresponds to a presupposition failure. An example of a derivation involving a definite is provided in Figure 41. Context is significant in nominal interpretation. In particular, the property contributed by the noun can be further restricted by context. Thus an utterance of *the receptionist* can uniquely determine a referent if the context restricts attention to the appropriate receptionist. This is exactly the same phenomena as occurs with quantified

$$\dfrac{\dfrac{\dfrac{\textit{which}}{q(n\backslash n/(s\uparrow np), np, np) \quad \lambda A.\lambda V.\lambda P.\lambda x.P(x)\wedge V(A(x))}\,l}{np\colon y}\,qe^0}{n\backslash n/(s\uparrow np)\colon \lambda V.\lambda P.\lambda x.P(x)\wedge V(x)}\,qe^0$$

Figure 38: Analysis of *which*

$$\dfrac{\dfrac{\textit{table}}{n \quad \mathbf{table}}\,l \qquad \dfrac{\dfrac{\dfrac{\dfrac{\textit{every}}{s\Uparrow np/n \quad \mathbf{every}}\,l \qquad \dfrac{\dfrac{\textit{leg of}}{n/np \quad \mathbf{leg}}\,l \qquad \dfrac{\dfrac{\textit{which}}{q(n\backslash n/(s\uparrow(s\Uparrow np)), s\Uparrow np, np) \quad \lambda A.\lambda V.\lambda P.\lambda x.P(x)\wedge V(A(x))}\,l}{np\colon y}\,qe^2}{n\colon \mathbf{leg}(y)}\,/e}{s\Uparrow np\colon \mathbf{every}(\mathbf{leg}(y))}\,/e}{n\backslash n/(s\uparrow(s\Uparrow np))\colon \lambda V.\lambda P.\lambda x.P(x)\wedge V(\mathbf{every}(\mathbf{leg}(x)))}\,qe^2 \qquad \dfrac{\textit{broke}}{s\uparrow(s\Uparrow np) \quad \lambda x_7.x_7(\mathbf{break})}\,d}{n\backslash n\colon \lambda P.\lambda x.P(x)\wedge \mathbf{every}(\mathbf{leg}(x))(\mathbf{break})}\,/e}{n\colon \lambda x.\mathbf{table}(x)\wedge \mathbf{every}(\mathbf{leg}(x))(\mathbf{break})}\,\backslash e$$

Figure 39: Analysis of *table every leg of which broke*

itself pied-piped. Such a case was not explicitly treated by Morrill, but follows directly from our theory of quantification by simply instantiating his polymorphic lexical entry. An example of quantifier pied-piping is provided in Figure 39. The key to this analysis is the derivation of *broke* as category $s\uparrow(s\Uparrow np)$, which follows the derivation in Figure 25. Otherwise, the derivation is similar to our previous derivation in Figure 37.

In examples such as the following, we derive a scope ambiguity between the quantifier being pied-piped and quantifiers within the sentence with a quantifier gap.

(41) the author every paper by whom some journal accepted

Here we get an ambiguity between whether or not it is the same journal accepting each of the author's papers. This affect is achieved due to the correct interaction of the gap introduction rules and the quantifier rule, as shown in the two analyses of *some journal accepted* in Figure 40. Note that these derivations simply parallel those in Figure 28. Depending on which of these analyses is used, different scopings of the pied-piped quantifier will be generated.

In addition to generating the correct readings for pied-piped quantifiers, our theory does not generate ill-formed analyses in which variables are unbound.

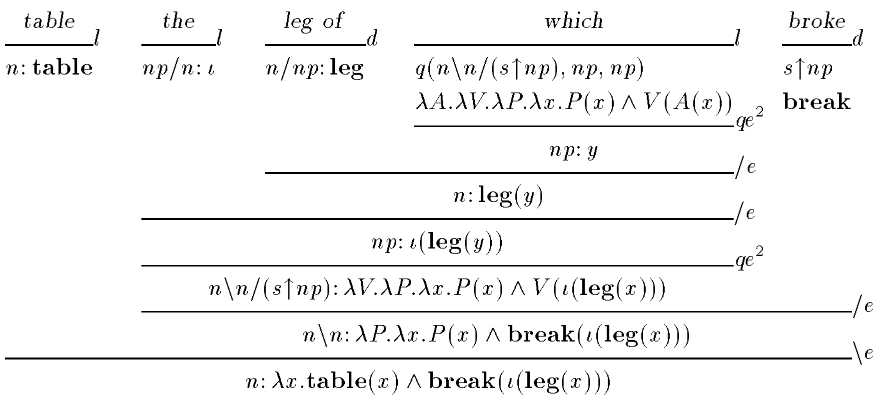

Figure 37: Analysis of *table the leg of which broke*

Morrill [1992b] has recently answered Pollard's [1988] challenge to provide an analysis of pied-piping in categorial grammar. His analysis exploits the q constructor, assigning the following category to relative pronouns.

(40) *which* $\Rightarrow q(n\backslash n/(s{\uparrow}A), A, np)\colon \lambda N.\lambda V.\lambda P.\lambda x.P(x) \wedge V(N(x))$
$[A \in \{np, s{\Uparrow}np, n\backslash n, s\backslash np\backslash(s\backslash np)\}]$

This is a polymorphic lexical entry, in that it is parameterized for values of the category A. The values for A indicate the categories which may be pied piped in English. Of course, we have to be careful to choose the appropriately typed variables for each instance of A. For our purposes, we may treat the above lexical entry as schematic rather than truly polymorphic. With A instantiated to np, we have pied-piping of a noun phrase. An analysis of a noun modified by a pied-piped relative clause is given in Figure 37. To focus on the important part of the analysis, we have suppressed the analysis of *leg of*, which involves slash introduction and type raising. Furthermore, we have abbreviated the analysis of *broke* as an $s{\uparrow}np$, which is carried out by $\uparrow$ introduction. This allows us to focus on the behavior of the relative pronoun *which*. Here it is treated as a noun phrase in the analysis of *the leg of which*, which is seeking a sentence with a noun phrase gap. The meaning of *the leg of which* semantically fills the gap in the sentence, with the variable introduced for *which* becoming identified with the nominal variable in the final analysis. Pied-piping of other categories, such as prepositional phrases, follows exactly the same pattern; the pied-piped element fills a gap in a sentence, with the relative pronoun's variable being identified with that of the noun being restricted by the relative pronoun.

Morrill notes with his analysis that the pied-piped category for relatives is more general than the standard one. In fact, we can derive our previous relative pronoun category as shown in Figure 38. This is just the boundary case of pied-piping, in which the only element pied-piped is the relative itself.

The interesting case for present purposes is that in which a quantifier is

q Left

$$\frac{\Gamma_1, C\colon x, \Gamma_2 \Rightarrow B\colon \beta}{\Gamma_1, q(A,B,C)\colon \alpha, \Gamma_2 \Rightarrow A\colon \alpha(\lambda x.\beta)}\, ql$$

$[x \text{ fresh}]$

q Right

$$\frac{}{A\colon \alpha \Rightarrow q(A,B,B)\colon \lambda P.P(\alpha)}\, qr$$

Figure 35: Sequent Schemes for q

q Elimination

$$\frac{\begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \dfrac{q(A,B,C)\colon \alpha}{C\colon x}\, qe^i & \vdots \\ \vdots & \vdots & \vdots \end{array}}{\dfrac{B\colon \beta}{A\colon \alpha(\lambda x.\beta)}\, qe^i}$$

q Introduction

$$\frac{A\colon \alpha}{q(A,B,B)\colon \lambda P.P(\alpha)}\, \Uparrow i$$

Figure 36: Natural Deduction Schemes for q

connective, these schemes are not complete for their intended interpretation, but this is not important in the current context.[20] With these rules we have the following logical equivalence.

(39) $A \Uparrow B\colon \alpha \equiv q(A,B,B)\colon \alpha$

Thus we can treat the binary scoping constructor $\Uparrow$ as an instance of the more general ternary scoping constructor q.

[20] Morrill and Solias [1992] are able to define q in terms of their constructors $\uparrow$ and $\downarrow$, by setting $q(A,B,C) = A\downarrow(B\uparrow C)$, a move suggested by Moortgat [1988], though it was unsound in his logic. Unfortunately, as we mentioned earlier, Morrill and Solias's approach is incomplete with respect to the intended interpretation of the quantification constructor [Morrill 1992a].

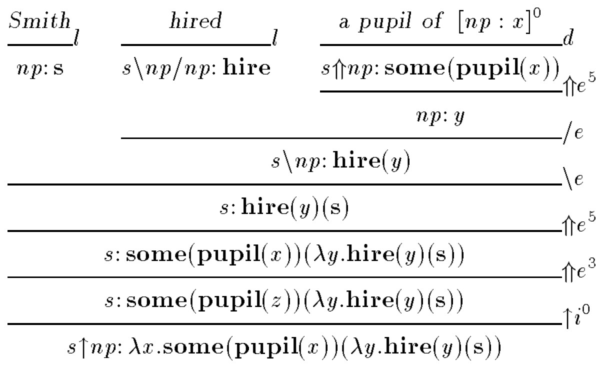

Figure 34: Analysis of *Smith hired a pupil of*

Our theory correctly characterizes the fundamental interaction between extraction and quantification, seen in examples such as (6)a. We provide another such example below.

(37) Every professor that [Smith hired [a pupil of __]] retired.

This example illustrates that quantifier internal gaps bound by relative pronouns can not take wide scope with respect to the quantifier binding the nominal in which they occur. Such an occurrence would lead to an unbound variable. The reason such an analysis is blocked in our system is again due to the logical structure of our derivations, which requires their proper nesting. The only (normal) derivation of the relative complement is shown in Figure 34. An attempt to give the nested quantifier any wider scope would not allow the relative complement to be derived, and would thus prohibit any derivation other than the correct one.

7.1. *Generalized Quantification and Pied-Piping*

Moortgat assigned the category $A \Uparrow B$ to expressions which could act as Bs in the analysis of an A, at which point they could apply semantically to produce an A. Noticing the two occurrences of A in this definition, he introduced a more general ternary constructor q. Expressions assigned to category $q(A, B, C)$ are able to act as Cs in the context of a derivation of B, at which point they reduce semantically to produce an A. Thus we have the following clause for q and its corresponding typing.

(38) a. $q(A, B, C) \in \mathsf{Cat}$ if $A, B, C \in \mathsf{Cat}$

b. $\mathit{Typ}(Q(A, B, C)) = (\mathit{Typ}(C) \rightarrow \mathit{Typ}(B)) \rightarrow \mathit{Typ}(A)$

The sequent and natural deduction rules for q are the obvious ones, which we present in Figure 35 and Figure 36. As with the rules for the binary scoping

$$
\dfrac{\dfrac{\textit{student}}{n\colon \mathbf{student}}\,l
\qquad
\dfrac{\dfrac{\textit{who}}{n\backslash n/(s\uparrow np)\colon \lambda P.\lambda R.\lambda x.P(x) \wedge R(x)}\,l
\quad
\dfrac{\textit{Fred believed dropped out yesterday}}{s\uparrow np\colon \lambda y.\mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(y)))(\mathbf{f})}\,d}{n\backslash n\colon \lambda R.\lambda x.\mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(x)))(\mathbf{f}) \wedge R(x)}\,/e}{n\colon \lambda x.\mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(x)))(\mathbf{f}) \wedge \mathbf{student}(x)}\,\backslash e
$$

Figure 32: Analysis of *student who Fred believed dropped out yesterday*

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textit{every kid}}{s{\Uparrow}np\colon \mathbf{every}(\mathbf{kid})}\,d}{np\colon y}\,{\Uparrow}e^2
\quad
\dfrac{\dfrac{\textit{likes}}{s\backslash np/np\colon \mathbf{like}}\,l \quad [np\colon x]^0}{s\backslash np\colon \mathbf{like}(x)}\,/e}{s\colon \mathbf{like}(x)(y)}\,\backslash e}{s\colon \mathbf{every}(\mathbf{kid})(\mathbf{like}(x))}\,{\Uparrow}e^2}{s\uparrow np\colon \lambda x.\mathbf{every}(\mathbf{kid})(\mathbf{like}(x))}\,{\uparrow}i^0
$$

Figure 33: Analysis of *every kid likes*

pronouns, which is the natural generalization of Steedman's [1985] semantics for the relative pronouns.

(36) *who* $\Rightarrow n\backslash n/(s\uparrow np)\colon \lambda P.\lambda R.\lambda x.P(x) \wedge R(x)$

In conjunction with the analysis of the relative complement in Figure 31, we are able to produce the derivation in Figure 32.

As with our previous analyses, now that we have characterized the logic for gaps, we derive a number of results which have previously been stipulated. Two such predictions are forthcoming in the case of relative clauses with embedded quantifiers. First, consider the example in (2)c. Here we can reduce the quantifier internally to the relative clause by applying slash introduction and then applying quantifier elimination, as shown in Figure 33. When the phrase derived here occurs as a complement to a relative clause, the result is narrow scope for the quantifier.

It has been observed that relative clauses are so-called *islands* for embedded quantifiers [Ross 1967; Postal 1974; Rodman 1976; Fodor and Sag 1982]. In other words, quantifiers can usually not take scope wider than the relative clause in which they appear. But the data is very subtle, and beyond the scope of the current paper. We do note, though, that Morrill [1990a, 1992b] has introduced a logical analysis of islands in terms of structural modalities. Morrill's goal was to account for islandhood with respect to extraction, but the same categories he used to block extraction could be used to block quantifiers from escaping.

Gap Introduction

$$\dfrac{\dfrac{\begin{matrix}\vdots & [A\colon x]^n & \vdots\\ \vdots & \vdots & \vdots\end{matrix}}{B\colon \alpha}}{B{\uparrow}A\colon \lambda x.\alpha}\,{\uparrow}i^n$$

Figure 30: Gap Natural Deduction Scheme

$$\dfrac{\dfrac{\dfrac{\textit{Fred}}{np\colon \mathbf{f}}\,l \quad \dfrac{\dfrac{\textit{believed}}{s\backslash np/s\colon \mathbf{bel}}\,l \quad \dfrac{[np\colon y]^0 \quad \dfrac{\dfrac{\textit{dropped out}}{s\backslash np\colon \mathbf{dropout}}\,l \quad \dfrac{\textit{yesterday}}{\begin{matrix}s\backslash np\backslash(s\backslash np)\\ \lambda V.\lambda x.\mathbf{yes}(V(x))\end{matrix}}\,l}{s\backslash np\colon \lambda x.\mathbf{yes}(\mathbf{dropout}(x))}\,\backslash e}{s\colon \mathbf{yes}(\mathbf{dropout}(y))}\,\backslash e}{s\backslash np\colon \mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(y)))}\,/e}{s\colon \mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(y)))(\mathbf{f})}\,\backslash e}{s{\uparrow}np\colon \lambda y.\mathbf{bel}(\mathbf{yes}(\mathbf{dropout}(y)))(\mathbf{f})}\,{\uparrow}i^0$$

Figure 31: Analysis of *Fred believed dropped out yesterday*

is in order to account for phrase-internal gaps, such as those occurring in the following.

(35) a. The student who [the professor met __ yesterday on the quad] filed a complaint.

b. The student who [Fred believed __ had dropped out] is back.

In both of these cases, the noun phrase gap, indicated as usual by an underscore, is internal to the phrase from which it is missing, indicated by bracketing. Both of these bracketed phrases will be analyzable as being of category $s{\uparrow}np$.[19] Such an analysis is shown in Figure 31, where for simplicity, we have employed a reduced entry for the adverbial *yesterday*. The modificational behavior of relative clauses arises from the assumption of the following category for relative

[19] This fact highlights one of the primary differences between logical approaches to grammar and those developed in the transformational tradition [Gazdar 1981a]. Transformational accounts typically treat the bracketed phrases in (35) as being rooted at category s, with the occurrence of an empty category as a leaf in the tree indicating that there is a gap. Ed Stabler [personal communication] has pointed out that these traditions could be brought closer together by thinking of the root s of the transformational derivation as being marked in addition with the categories remaining to be governed.

Gap Right

$$\frac{\Gamma_1,\ B{:}\,x,\ \Gamma_2 \Rightarrow A{:}\,\alpha}{\Gamma_1,\ \Gamma_2 \Rightarrow A{\uparrow}B{:}\,\lambda x.\alpha}{\uparrow}r$$

$[x \text{ fresh}]$

Figure 29: Gap Sequent Scheme

7. Unbounded Dependencies and Quantification

In this section, we turn our attention to a logical account of unbounded dependencies. After introducing the sequent and natural deduction versions of Moortgat's [1988] calculus, we will see how unbounded dependencies interact in the correct way with quantifiers. We further show how this conception of unbounded dependencies provides a logical analogue of the slash-passing mechanisms of GPSG [Gazdar 1981b].

Like quantifiers, unbounded dependencies are characterized by a binary constructor, which was originally introduced by Moortgat [1988]. Its form and type are as follows.

(34) a. $A{\uparrow}B \in \mathsf{Cat}$ if $A, B \in \mathsf{Cat}$

b. $Typ(A{\uparrow}B) = Typ(B) \rightarrow TypA$

The $\uparrow$ operator is interpreted in the same way as the slash constructor in GPSG. Specifically, an expression of category $A{\uparrow}B$ can be thought of as an incomplete A which is missing a B. In this section, we will focus on the category $s{\uparrow}np$, which is assigned to sentences with an np gap.[17]

Moortgat formulated only a right rule for the gap constructor, which we present in Figure 29.[18] Like the right rules for slashes, we require the variable x to be fresh and of the appropriate type. The natural deduction equivalent of the sequent scheme is given in Figure 30.

The gap sequent scheme is similar in nature to the right rules for the slashes; in fact, its semantics is identical. The only real difference is that the hypothetical category is not required to occur peripherally in the gap schemes. This

[17] Steedman's [1985] previous treatment of unbounded dependencies in categorial grammar attempted to do without a special gap constructor. Instead, he analyzed sentences with noun phrase gaps as either being of category s/np or $s\backslash np$. In HPSG [Pollard and Sag in press], the most fine-grained of the theories related to ours, gaps come in three flavors, depending on whether they will be used for wh-relatives, wh-questions or pure movement.

[18] Solias [1992, Morrill and Solias 1992], by keeping track of the point of insertion, derived a dual left rule for the use of a gapping constructor. In particular, she assigned the expression $\langle e_1, e_2\rangle$ to the category $A{\uparrow}B$ if and only if for every expression e_3 of category B, $e_1 \cdot e_2 \cdot e_3$ is assigned to category A. With this control over the point of insertion, the rule of use is the obvious one in which an expression of category $A{\uparrow}B$ wraps around an expression of category B at the point of insertion, to form a result of category A. She also introduced a dual constructor $\downarrow$, such that expressions of category $A{\downarrow}B$ could be inserted into categories of expression B, at a specified point of insertion, to produce a result of category A.

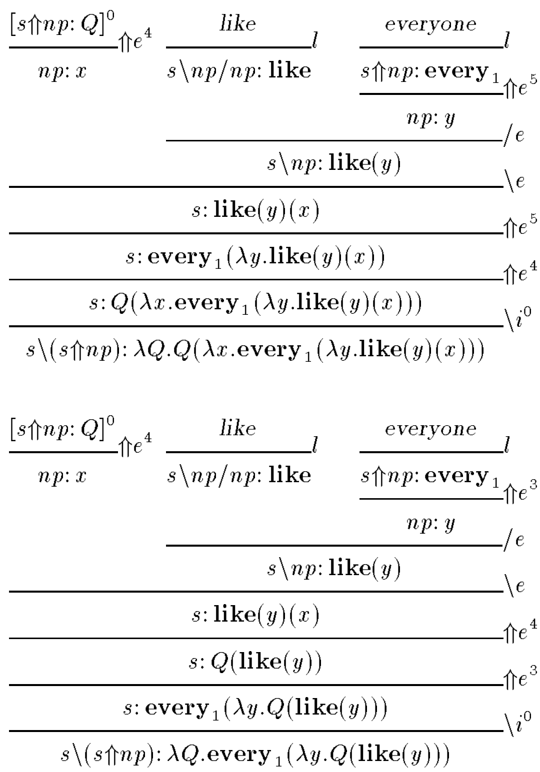

Figure 28: Analyses of *like everyone*

Note that these readings are derived without making the standard transformational assumption that the object and infinitive phrase form a constituent sentence under some kind of exceptional case marking. This greatly simplifies the treatment of the passive as applied to control verbs in general [Bresnan 1982, Pollard and Sag 1987, Carpenter 1992a].[16]

With the reduced lexical entries for "equi" verbs, the following examples display less ambiguity.

(33) a. John persuaded someone to help everyone.

b. Someone promised John to help everyone.

There are only three readings in each case: one in which the universal is inside the scope of the control verb, derived as above, and two in which both quantifiers are wide, with alternative scopes, derived in the usual way by quantifier elimination.

[16] A more general, polymorphic lexical entry for "raising" verbs would, in a uniform manner, allow the full range of controllers noted by Bresnan [1982], such as pleonastic *it* and *there*, inverted prepositional phrases, and so on. See [Emms 1990] and [Morrill 1992a] for suggestions concerning polymorphism.

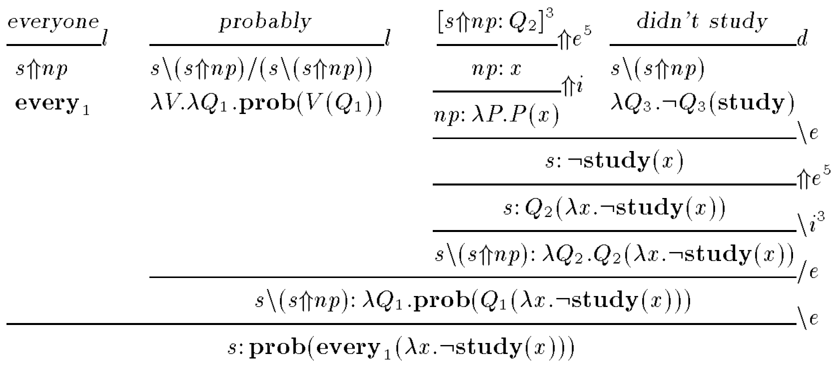

Figure 27: Analysis of *everyone probably didn't study*

For (3)f, where there is a quantifier in the object position of the controlled verb phrase, the slash introduction rule combined with the quantifier elimination rule allows us to derive the narrow scope reading of the embedded quantifier. Such an analysis for the case of (3)f is provided in the first derivation in Figure 28. For concreteness, we can follow GPSG [Gazdar et al. 1982] in treating the infinitive marker *to* as an auxiliary with the identity function as its semantics, just as in our treatment of *did* in (26)c. The remainder of the analysis of (3)f proceeds by slash elimination, after type raising the subject, to derive the following result.

(28) *John seems to like everyone* $\Rightarrow s\colon \mathbf{seem}(\mathbf{every}_1(\lambda x.\mathbf{like}(x)(\mathbf{j})))$

Of course, by simply using quantifier elimination on the embedded quantifier, we can derive the wide scope reading given below.

(29) *John seems to like everyone* $\Rightarrow s\colon \mathbf{every}_1(\lambda x.\mathbf{seem}(\mathbf{like}(x)(\mathbf{j})))$

There is also the possibility of reducing the quantifiers in Figure 28 in the opposite order, as given in the second derivation. This generates the following analysis.

(30) *like everyone* $\Rightarrow \lambda Q.\mathbf{every}_1(\lambda x.Q(\lambda y.\mathbf{like}(y)(x)))$

This allows us to derive a six way ambiguity in the following example of a subject control "raising" verb with a quantified subject and a quantified object in the controlled infinitival complement.

(31) Someone is guaranteed to like everyone.

Here there are two possibilities in which the quantifiers are internal to the guarantee, two in which they are external, and two in which one is internal and one is external. There are six analogous readings for object "raising" verbs, as in:

(32) John believes someone to like everyone.

$$\dfrac{\dfrac{\textit{everyone}}{s{\Uparrow}np\colon \mathbf{every}_1}\,l \qquad \dfrac{\dfrac{\textit{didn't}}{s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q_1.\neg V(Q_1)}\,l \qquad \dfrac{\textit{study}}{s\backslash(s{\Uparrow}np)\colon \lambda Q_2.Q_2(\mathbf{study})}\,l}{s\backslash(s{\Uparrow}np)\colon \lambda Q_1.\neg Q_1(\mathbf{study})}\,/e}{s\colon \neg\mathbf{every}_1(\mathbf{study})}\,\backslash e$$

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\textit{everyone}}{s{\Uparrow}np\colon \mathbf{every}_1}\,l}{np\colon x}\,{\Uparrow}e^0}{np\colon \lambda P.P(x)}\,{\Uparrow}i \qquad \dfrac{\dfrac{\textit{didn't}}{s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q_1.\neg V(Q_1)}\,l \qquad \dfrac{\textit{study}}{s\backslash(s{\Uparrow}np)\colon \lambda Q_2.Q_2(\mathbf{study})}\,l}{s\backslash(s{\Uparrow}np)\colon \lambda Q_1.\neg Q_1(\mathbf{study})}\,/e}{s\colon \neg\mathbf{study}(x)}\,\backslash e}{s\colon \mathbf{every}_1(\lambda x.\neg\mathbf{study}(x))}\,{\Uparrow}e^0$$

Figure 26: Analysis of *everyone didn't study*

The wide scope reading of the quantifier is achieved by a combination of the quantifier elimination and introduction schemes.

An analysis such as that in Figure 26 has traditionally not been a problem for storage-based accounts or quantifying-in accounts.[15] On the other hand, the case where control examples nest, such as (3)b have been. While the wide scope and narrow scope readings are generated just as in Figure 26, the intermediate readings have proven to be problematic. Here is another case for which our categorial logic pays off. The interaction between the slash schemes and the quantifier schemes captures the intermediate reading, as shown in Figure 27.

Cases of raising and equi-type control verbs can be handled in exactly the same fashion. The lexical entries required for the remaining examples in (3), (3)c through (3)f, are as follows.

(27) a. *seems* $\Rightarrow s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q.\mathbf{seem}(Q(V))$

b. *believes* $\Rightarrow s\backslash np/(s\backslash(s{\Uparrow}np))/(s{\Uparrow}np)\colon \lambda Q.\lambda V.\lambda x.\mathbf{bel}(V(Q))(x)$

c. *persuaded* $\Rightarrow s\backslash np/(s\backslash np)/np\colon \lambda y.\lambda V.\lambda x.\mathbf{persuade}(V(y))(y)(x)$

With these lexical entries, the analyses of (3)c through (3)e are derived by simple slash elimination. The distinction between so-called "equi" and "raising" control verbs is determined lexically; the "equi" verbs, such as *persuade*, are controlled by simple noun phrases rather than by quantifiers. This forces wide scope, as there is no way to raise a noun phrase argument to a quantifier with narrow scope.

[15] Except for those accounts such as found in HPSG [Pollard and Sag in press], where quantifiers always begin in storage.

$$\dfrac{\dfrac{\dfrac{\dfrac{[s{\Uparrow}np\colon Q]^0}{np\colon x}{\Uparrow}e^2 \qquad \dfrac{\mathit{runs}}{s\backslash np\colon \mathbf{run}}l}{s\colon \mathbf{run}(x)}\backslash e}{s\colon Q(\mathbf{run})}{\Uparrow}e^2}{s\backslash(s{\Uparrow}np)\colon \lambda Q.Q(\mathbf{run})}\backslash i^0$$

Figure 25: Type Raised Analysis of *runs*

6. Control and Quantification

In this section, we provide the lexical entries necessary to capture the correct pattern of behavior displayed by control constructions. In categorial grammars, control is effected syntactically by categorizing control verbs as taking verb phrase complements [Steedman 1988]. Semantically, control is achieved by β-reduction of shared variables. In this way, many disparate categories, such as adverbials, negative particles, auxiliary verbs and the usual control verbs, are treated identically from a syntactic vantage point.[13]

Consider the following lexical entries:

(26) a. *not* $\Rightarrow s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q.\neg V(Q)$

b. *probably* $\Rightarrow s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q.\mathbf{prob}(V(Q))$

c. *did* $\Rightarrow s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q.V(Q)$

d. *didn't* $\Rightarrow s\backslash(s{\Uparrow}np)/(s\backslash(s{\Uparrow}np))\colon \lambda V.\lambda Q.\neg V(Q)$

All of these categories take verb phrases seeking quantified subjects as arguments and result in the same kind of category. The only difference is in semantic effect.[14] Note that the category assigned to *didn't* is derivable by slash introduction as an analysis of *did not*.

From our minimal category assignment for verb phrases, $s\backslash np$, we show in Figure 25 how to derive the category $s\backslash(s{\Uparrow}np)$ by slash introduction and quantifier elimination. In Figure 26, we show the two analyses for an ambiguous sentence such as (3)a, in which the subject is quantified and the verb is negated. The narrow scope reading of the quantifier with respect to the negation, corresponding to the first analysis in Figure 26, is derived naturally by application.

[13] This is more or less true in LFG [Bresnan 1982] and HPSG [Pollard and Sag 1987, in press, Sag and Pollard 1991] approaches to control, though both of these theories allow finer grained syntactic distinctions to be made than we allow for here. The use of unification in these theories achieves an effect similar to that of β-reduction in the categorial approach.

[14] For the sake of simplicity, we ignore the issue of tense and of syntactic marking, both of which can be incorporated straightforwardly. For tense, the approach of Hinrichs [1988] is especially compatible with our approach. In terms of the syntactic marking of auxiliaries, we adopt the GPSG approach of Gazdar, Pullum and Sag [1982], which has been worked out in detail for the categorial setting by Carpenter [1992a].

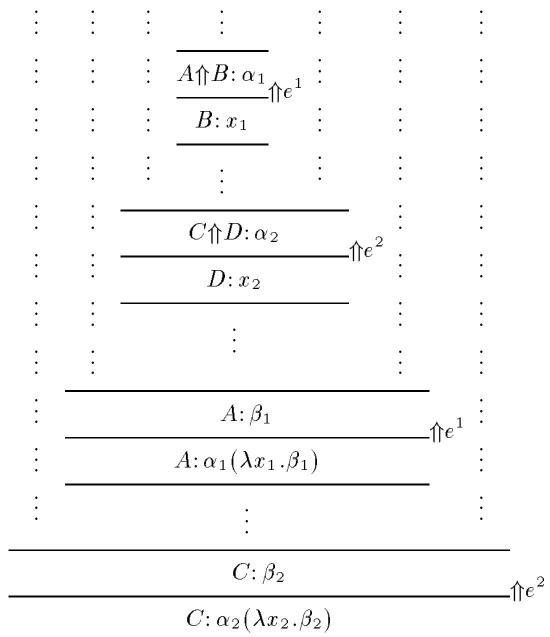

Figure 24: Scoping Restriction Violation

conclusion. The proper interpretation of the natural deduction schemes is identical. In particular, in Figure 24, $A\colon \beta_1$ is not properly derived as there is still an undischarged hypothetical assumption. Hence the attempt to introduce the quantifier α_1 and eliminate the hypothesis of $B\colon x_1$ is blocked.

To correct this problem suffered by Cooper's approach, a number of moves have been taken. In HPSG [Pollard and Sag in press], for example, a principle of quantifier binding is enforced that effectively blocks derivations in which all variables are not bound. This is similar to the effects achieved by a similar restriction in the system of Hobbs and Shieber [1987], which shares properties of both storage and quantifying-in. A logically more natural approach in which the store is structured as a nested list was independently introduced by Keller [1988] and Gerdemann and Hinrichs [1990]. In retrospect, it can be seen that their nesting mechanisms are nothing more than a "semantic" encoding of the structural condition on derivations imposed by our quantificational logic. Furthermore, our approach explains why storage mechanisms, which seem so closely related to Montague's quantifying-in rule, originally derived different results — the full logical structure of Montague's approach was not encoded in the phrase-structure derivations of Cooper.

$$\frac{\textit{student}}{n\colon \mathbf{student}}\,l \qquad \frac{\textit{in}}{n\backslash n/(s\Uparrow np)\ \ \lambda Q.\lambda P.\lambda x.P(x)\wedge Q(\lambda y.\mathbf{in}(y)(x))}\,l \qquad \frac{\textit{every class}}{s\Uparrow np\colon \mathbf{every}(\mathbf{class})}\,d$$

$$\frac{n\backslash n/(s\Uparrow np)\ \ \lambda Q.\lambda P.\lambda x.P(x)\wedge Q(\lambda y.\mathbf{in}(y)(x)) \qquad s\Uparrow np\colon \mathbf{every}(\mathbf{class})}{n\backslash n\colon \lambda P.\lambda x.P(x)\wedge \mathbf{every}(\mathbf{class})(\lambda y.\mathbf{in}(y)(x))}\,/e$$

$$\frac{n\colon \mathbf{student} \qquad n\backslash n\colon \lambda P.\lambda x.P(x)\wedge \mathbf{every}(\mathbf{class})(\lambda y.\mathbf{in}(y)(x))}{n\colon \lambda x.\mathbf{student}(x)\wedge \mathbf{every}(\mathbf{class})(\lambda y.\mathbf{in}(y)(x))}\,\backslash e$$

Figure 23: Analysis of *student in every class*

quantifiers in order to act as complements of prepositions. This is analgous to the treatment of simple noun phrases as complements to intensional verbs. For instance, by type raising $np\colon \mathbf{chi}$ to $s\Uparrow np\colon \lambda P.P(\mathbf{chi})$, we are able to derive:

(23) *in Chicago* $\Rightarrow n\backslash n\colon \lambda P.\lambda x.P(x)\wedge \mathbf{in}(\mathbf{chi})(x)$

We also adopt a type-raised account of modifying prepositions, with the following kind of lexical assignments:

(24) *in* $\Rightarrow n\backslash n/(s\Uparrow np)\colon \lambda Q.\lambda P.\lambda x.P(x)\wedge Q(\lambda y.\mathbf{in}(y)(x))$

An example of a derivation of a nominal with a quantifier embedded in a preposition is provided in Figure 23. This derivation contributes the nominal for the narrow scope reading of (2)a. The wide scope reading is derived as usual, by allowing the embedded quantifier to take sentential scope, using a combination of elimination and introduction, following the same pattern as the relational noun analysis in Figure 22.

As before, our analysis is strikingly similar in its sequent form to Montague's, and in its natural deduction form to Cooper's. Our quantifier elimination is analogous to term insertion and our quantifier introduction to type raising. Like Montague, we enjoy the property of not being able to derive unbound variables in our representations. Using Cooper's quantifier storage mechanism [1982, 1983], there was no way to block the derivation of the following erroneous result:

(25) *a student in every class failed* $\Rightarrow$
$s\colon \mathbf{some}(\lambda x.\mathbf{student}(x)\wedge \mathbf{in}(y)(x))(\lambda x.\mathbf{every}(\mathbf{class})(\lambda y.\mathbf{fail}(x)))$

The fault in Cooper's derivation lies in the unbound occurrence of the variable y in the restriction of the existential determiner. The fault in his system is the lack of a nesting restriction on quantifier retrieval. Under our approach, we can not perform such derivations, which take the general form shown in Figure 24. Such non-derivations are blocked because of the way in which we interpret our quantifier elimination schemes as natural deduction equivalents of the left sequent scheme for quantifiers. It is clear from the sequent presentation in Figure 9 that it is impossible to derive results in which variables are unbound. Every variable introduced in the antecedent sequent is bound in the

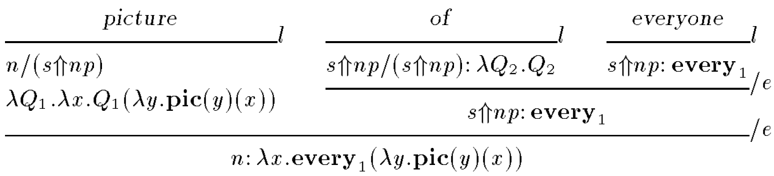

Figure 21: Analysis of *picture of everyone*

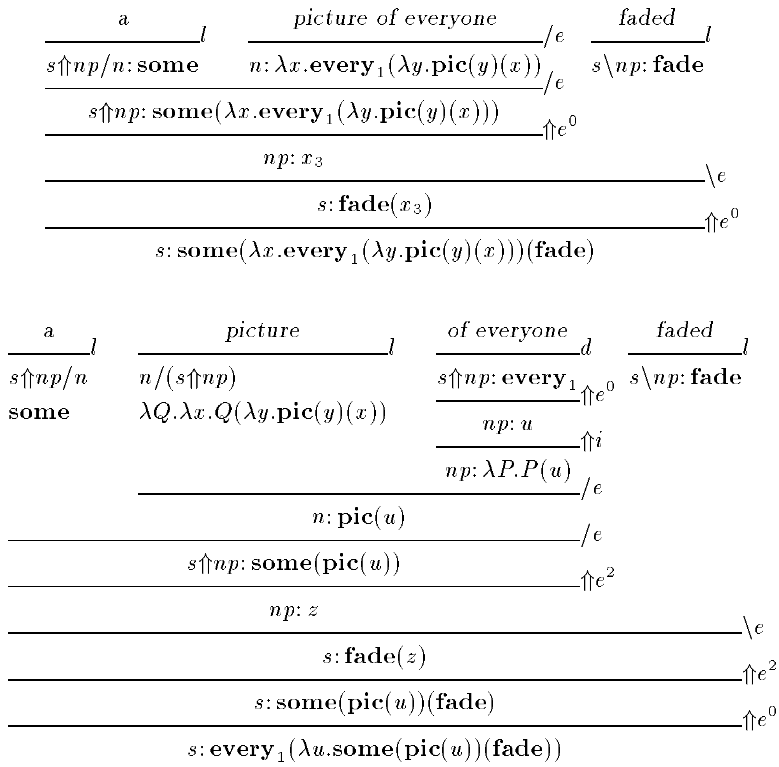

Figure 22: Analysis of *a picture of everyone faded*

to one analysis of an example like (2)b, as shown in the first part of Figure 22. But embedded quantifiers are also allowed to take wide scope. As usual in a Montagovian approach, this is achieved by using the quantifying-in rule coupled with type raising, as in the second analysis in Figure 22. Note that we have normalized λ-terms; for instance, we use **fade**, which is the η-reduced form of $\lambda z.\mathbf{fade}(z)$, and $\mathbf{pic}(u)$, which is the η-reduced form of $\lambda x.\mathbf{pic}(u)(x)$, which itself is the doubly β-reduced form of $(\lambda Q.\lambda x.Q(\lambda y.\mathbf{pic}(y)(x)))(\lambda P.P(u))$.

It is useful to note at this point that noun phrases can simply be raised to

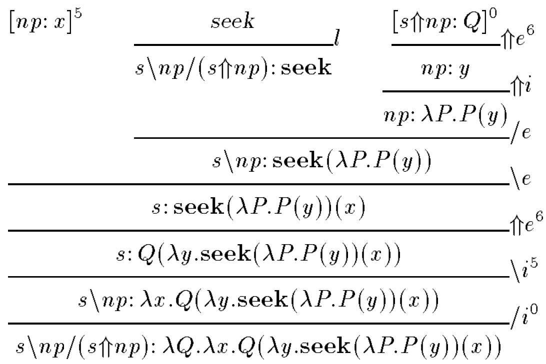

Figure 19: De Re Analysis of *seek*

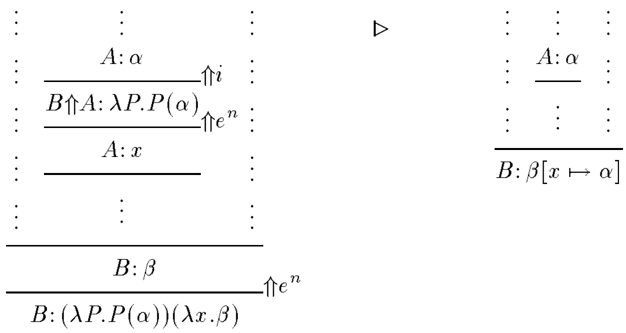

Figure 20: Scoping Normalization

5. Quantification in Nominals

In this section, we consider the role of quantifiers occurring within complex nominals. The two simple cases, represented by (2)a and (2)b, involve prepositional phrases, both as modifiers and complements to nouns. We first consider the relational noun case. Nouns such as *picture* receive lexical entries which are subcategorized for quantified, prepositionally-marked complements. Following GPSG [Gazdar et al. 1985], we assume that prepositions such as *of* syntactically function as case markers, and semantically as identity functions.

(22) a. *picture* $\Rightarrow n/(s{\Uparrow}np)\colon \lambda Q.\lambda x.Q(\lambda y.\mathbf{pic}(y)(x))$

b. *of* $\Rightarrow s{\Uparrow}np/(s{\Uparrow}np)\colon \lambda Q.Q$

We will not be concerned with the distribution of syntactic features here (but see Morrill [1992a]). In the simplest case, these lexical entries allow quantifiers to take scope within the nominal, as in Figure 21. This derivation contributes

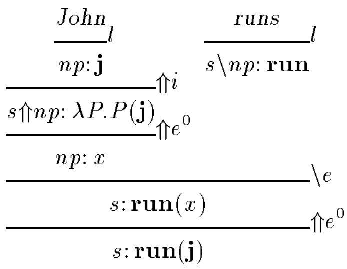

Figure 17: “Spurious” Analysis of *John runs*

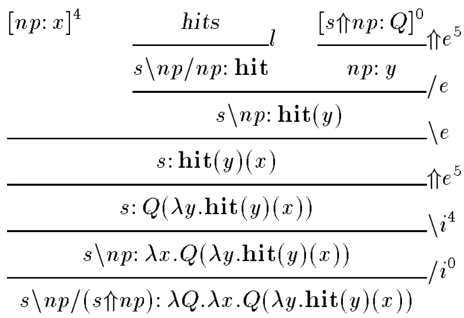

Figure 18: Derivation of Montague's Lexical Entry for *hits*

non-normal proof in Figure 17. Here we have raised the noun phrase subject to a quantifier and then reduced it. A similar example allows us to derive Montague's categories for transitive verbs, which always involve quantificational objects, from our simpler lexical assignment, as shown in Figure 18. Thus we avoid the lexical assignment of higher-order types where they are not necessary, but still enjoy the ability to derive the corresponding raised types when necessary. For instance, consider the coordination of an intensional and non-intensional transitive verb.

(21) The professor was seeking and later found a reference to type raising.

In our theory, such examples are predicted to be ambiguous; the seeking could be de dicto or de re, but of course the finding must be de re. This is because *seek* can be analyzed as in Figure 19.

We also avoid the introduction of meaning postulates for non-intensional verbs. Rather, we derive the same results from the normalization scheme for quantifier introduction and elimination. This scheme is given in Figure 20. The only thing to note is that the resulting semantic assignments are equivalent by two applications of β-reduction.

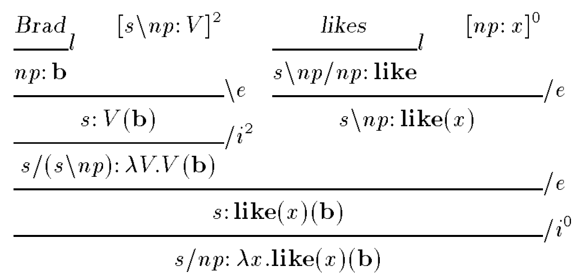

Figure 15: Analysis of *Brad likes*

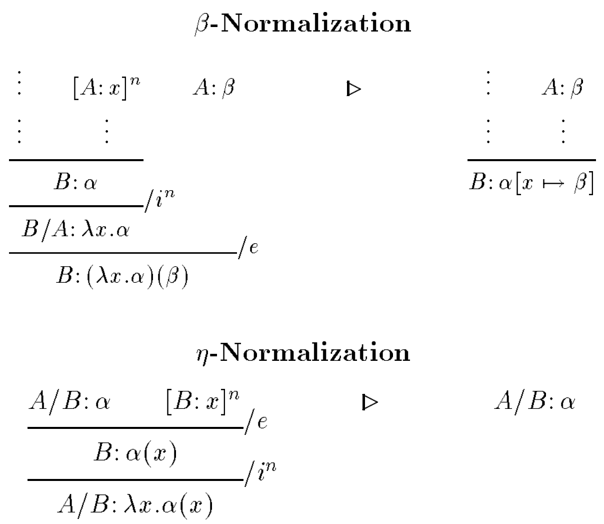

Figure 16: Slash Normalization

ization schemes illustrate the structural correspondence in the Curry-Howard morphism; the normalization of proofs corresponds to the normalization of λ-terms. Of course, there are analogous normalization schemes for the backward slash. By applying normalization rules recursively to subproofs, the end result is what is known as a *normal proof*. In general, such a proof involves as little use of introduction rules as is possible to derive the sequent in question. Such normal forms of categorial grammar proofs have been exploited in computational applications of Lambek's categorial grammars and their extensions by Hepple and Morrill [1989, Hepple 1990b, 1992] and König [1989], and to deal with similar problems of spurious ambiguity in parsing Steedman's combinatory categorial grammars by Hepple [1987] and Wall and Wittenburg [1989].

Just as we can normalize proofs involving slashes, we can normalize those involving combinations of type raising and scoping. For instance, consider the

$$\frac{\textit{John}}{np\colon \mathbf{j}}\,l \qquad \frac{\textit{sought}}{s\backslash np/(s{\Uparrow}np)\colon \mathbf{seek}}\,l \qquad \frac{\textit{a job}}{s{\Uparrow}np\colon \mathbf{some}(\mathbf{job})}\,/e$$

$$\frac{s\backslash np/(s{\Uparrow}np)\colon \mathbf{seek} \quad s{\Uparrow}np\colon \mathbf{some}(\mathbf{job})}{s\backslash np\colon \mathbf{seek}(\mathbf{some}(\mathbf{job}))}\,/e$$

$$\frac{np\colon \mathbf{j} \quad s\backslash np\colon \mathbf{seek}(\mathbf{some}(\mathbf{job}))}{s\colon \mathbf{seek}(\mathbf{some}(\mathbf{job}))(\mathbf{j})}\,\backslash e$$

$$\frac{\textit{John}}{np\colon \mathbf{j}}\,l \qquad \frac{\textit{sought}}{s\backslash np/(s{\Uparrow}np)\colon \mathbf{seek}}\,l \qquad \frac{\textit{a job}}{s{\Uparrow}np\colon \mathbf{some}(\mathbf{job})}\,d$$

$$\frac{s{\Uparrow}np\colon \mathbf{some}(\mathbf{job})}{np\colon x}\,{\Uparrow}e^0 \qquad \frac{np\colon x}{s{\Uparrow}np\colon \lambda P.P(x)}\,{\Uparrow}i$$

$$\frac{s\backslash np/(s{\Uparrow}np)\colon \mathbf{seek} \quad s{\Uparrow}np\colon \lambda P.P(x)}{s\backslash np\colon \mathbf{seek}(\lambda P.P(x))}\,/e$$

$$\frac{np\colon \mathbf{j} \quad s\backslash np\colon \mathbf{seek}(\lambda P.P(x))}{s\colon \mathbf{seek}(\lambda P.P(x))(\mathbf{j})}\,\backslash e$$

$$\frac{s\colon \mathbf{seek}(\lambda P.P(x))(\mathbf{j})}{s\colon \mathbf{some}(\mathbf{job})(\lambda x.\mathbf{seek}(\lambda P.P(x))(\mathbf{j}))}\,{\Uparrow}e^0$$

Figure 13: Analysis of *John sought a job*

$$\frac{\textit{brad}}{np\colon \mathbf{b}}\,l \qquad \frac{\textit{likes}}{s\backslash np/np\colon \mathbf{like}}\,l \qquad [np\colon x]^0 \qquad \frac{\textit{Pittsburgh}}{np\colon \mathbf{pgh}}\,l$$

$$\frac{s\backslash np/np\colon \mathbf{like} \quad [np\colon x]^0}{s\backslash np\colon \mathbf{like}(x)}\,/e$$

$$\frac{np\colon \mathbf{b} \quad s\backslash np\colon \mathbf{like}(x)}{s\colon \mathbf{like}(x)(\mathbf{b})}\,\backslash e$$

$$\frac{s\colon \mathbf{like}(x)(\mathbf{b})}{s/np\colon \lambda x.\mathbf{like}(x)(\mathbf{b})}\,/i^0$$

$$\frac{s/np\colon \lambda x.\mathbf{like}(x)(\mathbf{b}) \quad np\colon \mathbf{pgh}}{s\colon \mathbf{like}(\mathbf{pgh})(\mathbf{b})}\,/e$$

Figure 14: Analysis of *brad likes Pittsburgh*

many other ways to derive the same sentence. A second way in which the subject could first combine with the object is by raising the subject to the category assigned subjects by Steedman [1988], as shown in Figure 15. But there is a well-defined sense in which these derivations are less primitive than the simple ones using only application. We can define the notion of *proof normalization*, as it is used in other natural deduction calculi [Girard, Lafont and Taylor 1987]. Normalization rules reduce derivations of a sequent to simpler derivations of the same sequent. The normalization schemes for the forward slash are given in Figure 16. The normalization schemes eliminate an instance of slash introduction that is coupled with a corresponding slash elimination. It follows from the soundness of the β-conversion and η-conversion schemes in the λ-calculus that a normalized derivation produces the same semantic result. The normal-

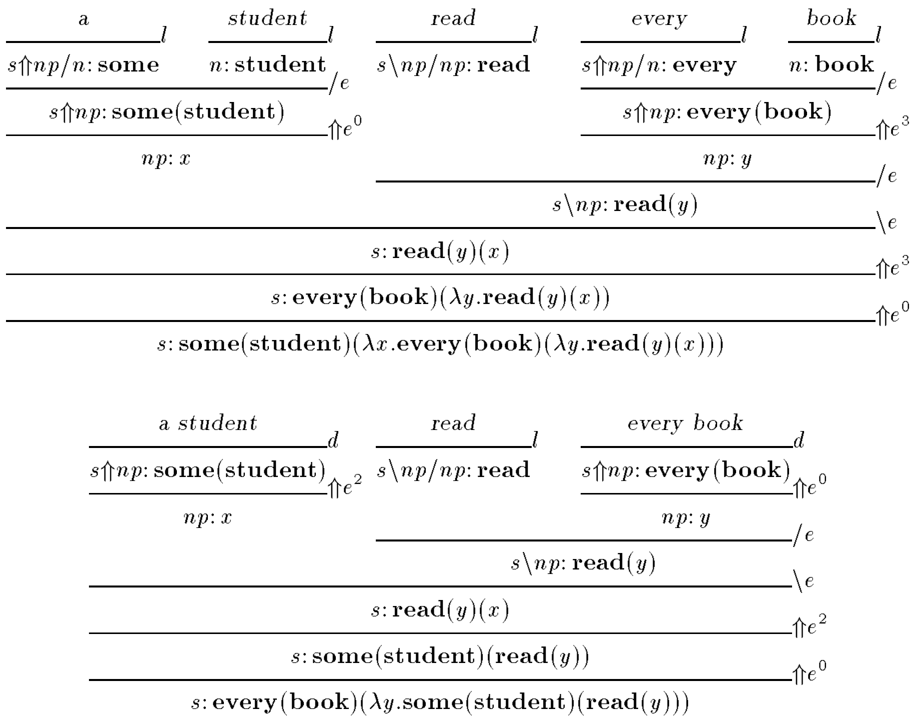

Figure 12: Analysis of *a student read every book*

version of (1)c are given in Figure 13. The first analysis corresponds to the de dicto case, in which John stands in the seeking relation to the generalized quantifier *some job*. The second analysis is for the de re case, in which there is a particular job x for which John stands in the seeking relation to the generalized quantifier $\lambda P.P(x)$ representing the properties of the job x.

4. "Spurious" Ambiguity and Normal Proofs

Our logic, in both natural deduction and sequent form, admits infinitely many different proofs of every provable sequent. For instance, abstractions can be carried out and then re-applied, or arbitrary categories can be type raised and then quantified-in. Such multiple derivations has often been referred to as *spurious ambiguity*, because the ambiguities in derivations do not correspond to ambiguities in assigned meanings. For instance, consider the redundant derivation of a sentence with a simple transitive verb in Figure 14. Here we have first combined the subject with the verb phrase, which requires the postulation of a hypothetical object. This illustrates the way in which such "spurious" ambiguities might be considered useful for left to right parsing; the hypothesized categories correspond to top-down predictions of material to follow. There are

Scope Elimination

$$\dfrac{\dfrac{\begin{array}{ccc} \vdots & \vdots & \vdots \\ \vdots & \dfrac{\dfrac{A{\Uparrow}B\colon \alpha}{B\colon x}{\Uparrow}e^i}{} & \vdots \\ \vdots & \vdots & \vdots \end{array}}{A\colon \beta}}{A\colon \alpha(\lambda x.\beta)}{\Uparrow}e^i$$

Scope Introduction

$$\dfrac{B\colon \beta}{A{\Uparrow}B\colon \lambda x.x(\beta)}{\Uparrow}i$$

Figure 11: Natural Deduction Scoping Schemes

noting generalized determiners, **every** and **some**, to the determiners of category $s{\Uparrow}np/n$. In this derivation and in following ones, we have normalized the λ-terms. For instance, we have used the η-reduced $\mathbf{read}(y)$ instead of the non-normal $\lambda x.\mathbf{read}(y)(x)$. Finally, we will shorten our derivations as much as possible by the use of derived rule schemes, which we mark with d. For instance, in the second derivation in Figure 12, we do not repeat the derivations of the generalized quantifiers.

The natural deduction form of the quantifier elimination schemes highlights the striking similarity between our logical approach and the phrase structure approach embodied in Cooper's [1982, 1983] storage mechanism The point at which a quantifier is eliminated and a hypothetical assumption is made is analogous to the point at which a quantifier is placed in storage. The point at which the assumption is discharged is analgous to the point at which the quantifier is removed from storage and applied semantically. So far, the only distinction seems to be that the intermediate nodes in our derivation do not carry information concerning the quantifiers in storage. Instead, this relationship is mediated by the quantifier elimination scheme. In retrospect, the way in which our sequent presentation corresponds to Montague's quantifying-in rule, and the way in which our natural deduction presentation of the same logical system correspond to Cooper storage, illustrate the deep logical connection between Cooper storage and quantifying-in — Cooper storage is an attempt at providing a natural deduction version of Montague's logic. We see below the ways in which this attempt ultimately failed, by not capturing the logical structure of Montague's quantifying-in scheme.

We can also adapt Montague's analysis of verbs such as *seek*, by treating them as taking generalized quantifiers as objects. Two analyses of a simplified

$$\dfrac{\dfrac{}{np\colon x,\ s\backslash np\colon \mathbf{run} \Rightarrow s\colon \mathbf{run}(x)}\backslash l}{s\Uparrow np\colon \mathbf{every}_1,\ s\backslash np\colon \mathbf{run} \Rightarrow s\colon \mathbf{every}_1(\lambda x.\mathbf{run}(x))}\Uparrow l$$

Figure 10: Example of Scoping

lexically assigned $s\Uparrow np\colon \mathbf{every}_1$. Here we take the semantic constant $\mathbf{every}_1$ to be the standard universal generalized quantifier of type $(\mathsf{Ind}{\rightarrow}\mathsf{Prop}){\rightarrow}\mathsf{Prop}$. The intended interpretation of $\mathbf{every}_1$ is as a function which maps a property to a true proposition if and only if the property is itself true of every individual.

The form of the left rule for quantifiers is very similar, both in spirit and in application, to Montague's quantifying-in rule. Like Montague's rule, we are able to combine an analysis involving a noun phrase with a variable semantics, and a quantifier, to produce a derivation where the quantifier binds the variable. The only real difference is that the notation of Montague, including construction specific schemes, as well as cumbersome and ill-motivated expressions such as he_n, is replaced with a logical scheme. Montague avoided the equivalent of sequent rules with antecedents, using instead a purely axiomatic system.

The right quantifier rule appears to be nothing more than Montague's lexical approach to type raising. But the logical presentation shows how the two rules are related to one another as duals. The right scoping rule allows us to perform type raising of the usual kind, as in the following derivable sequent.

(20) $np\colon \mathbf{j} \Rightarrow s\Uparrow np\colon \lambda P.P(\mathbf{j})$

This allows us to lexically assign proper names to the syntactic category np, and derive the generalized quantifier category. This avoids the tendency of Montague to generalize to the worst case for each construction and lexical entry, a strategy requiring further meaning postulates to avoid unwanted ambiguity. In general, we will assign the lowest adequate type to an expression and simply derive the raised types when necessary.

Before presenting further derivations, we present the natural deduction version of the scoping schemes in Figure 11. The introduction scheme is straightforward, as it is just the phrase structure analogue of type raising. The elimination scheme involves hypothetical reasoning; if $A\Uparrow B\colon \alpha$ can be derived, then we can treat it as being of category $B\colon x$ in the derivation of $A\colon \alpha$, at which point it takes semantic scope. The coordinated points in the derivation are indexed by the same integer. Note that the natural deduction scheme is simply a reformulation of the left sequent scheme, with an implicit use of the cut rule. Also note that there is no assumption that the quantifier appear on the periphery of the derivation, as with the forward and backward slash introduction schemes.

The two derivations of (1)a are shown in Figure 12. These derivations show that the order in which quantifiers are discharged determines their relative scopes. While we are not concerned with the logical interpretations of quantifiers, we note that in these derivations we have assigned constants de-

Scope Left

$$\frac{\Gamma_1,\ B\colon x,\ \Gamma_2 \Rightarrow A\colon \beta}{\Gamma_1,\ A{\Uparrow}B\colon \alpha,\ \Gamma_2 \Rightarrow A\colon \alpha(\lambda x.\beta)}\ {\Uparrow}r$$

$[x \text{ fresh}]$

Scope Right

$$\frac{}{B\colon \beta \Rightarrow A{\Uparrow}B\colon \lambda x.x(\beta)}\ {\Uparrow}l$$

Figure 9: Sequent Presentation of Scoping Schemes

investigated by Partee and Rooth [1983, Partee 1987]. But Hendriks' account is problematic in that it both overgenerates and undergenerates, as well as unduly complicating with nonlogical, recursive, asymmetric rule schemes, an otherwise elegant logical formalism.

In this section, we introduce Moortgat's scoping constructor, along with its introduction and elimination rules. The basic operation is similar to that of the term insertion rule employed by Montague. Moortgat's binary constructor $\Uparrow$ allows us to freely construct categories according to the following scheme:

(18) $A{\Uparrow}B \in \mathsf{Cat}$ if $A, B \in \mathsf{Cat}$

We assign a semantic type to a category of the form $A{\Uparrow}B$ by:

(19) $Typ(A{\Uparrow}B) = (Typ(B) \rightarrow Typ(A)) \rightarrow Typ(A)$

Our intended interpretation of quantifiers is such that an expression is assigned to the category $A{\Uparrow}B$ if and only if it can act as a B in the derivation of an A, at which point it can be applied semantically. Thus we assign the category $s{\Uparrow}np$ to generalized quantifiers; they act as noun phrases, and take semantic scope within embedding sentences.

As with other constructors, the rule schemes for the scoping constructor form a dual pair, with one left rule and one right rule. We present these rules in Figure 9. The left scoping rule is hypothetical in nature and corresponds to the use of a quantifier. The right rule corresponds to type raising.[12]

An example of the application of the left scoping rule can be found in Figure 10. This derivation is of the sentence *everyone ran*, where *everyone* is

[12] Unlike the slash rules, the pair of scoping rules are sound, but not complete with respect to their intended interpretation. For the slash rules, we can derive $e \Rightarrow A/B$ if and only if for every expression e' of category B, $e \cdot e'$ is of category A. In the case of scoping, we would like to be able to derive $e \Rightarrow A{\Uparrow}B$ if and only if e can occur as a B in the context of an A. Not every expression that can so occur will be analyzed as having category $A{\Uparrow}B$. Morrill and Solias [1992] have presented a more general logic for quantification, based in part on Montague's Universal Grammar [1970a] approach to categorial grammar, which extends the rules of proof for $\Uparrow$. And while it goes further than our logic, it is still not complete with respect to our intuitive construal of the quantification constructor.

to Lambek's calculus.[11]

(16) $$\frac{A\colon \alpha,\ B\colon x \Rightarrow D\colon \gamma}{A\colon \alpha,\ B/C\colon \beta \Rightarrow D/C\colon \lambda x.\gamma}\,gc/$$

Of course, the dual backward slash versions of generalized composition and type raising are necessary to achieve completeness. It is worth noting that generalized composition provides the logical perspective from which to understand the slash passing mechanism of GPSG [Gazdar 1981b]. In GPSG, such schemes were known as *metarules* in that they were used to generate a number of derived phrase structure schemes as instances.

But Steedman did not stop with so-called *harmonic* instances of the composition schemes, nor did those who followed him such as Moortgat [1988]. In addition, Steedman adopted *disharmonic* instances of composition, such as that given below.

(17) $A/B\colon \alpha,\ B\backslash C\colon \beta \Rightarrow A\backslash C\colon \lambda x.\alpha(\beta(x))$ [Disharmonic Composition]

Such a scheme was required to account for non-peripheral extractions. It was also adopted by Steedman [1985] to account for flexible word-order in languages such as Dutch. Adding even one disharmonic scheme to Lambek's calculus results in a permutation closed variant of Lambek's calculus [van Benthem 1986a]. To avoid this undesirable collapsing of generative power, as well as to capture other syntactic restrictions on distribution such as island constraints, Steedman only employed a limited number of instances of these schemes. Rather than restricting our schemes in such an asymmetric fashion, we adopt Lambek's schemes in their full generality. To account for unbounded dependencies, we adopt Moortgat's [1988] gapping constructor, which is endowed with its own inference schemes to allow for non-peripheral gaps. To account for free word order, modal constructors have been employed with a logic that allows them to be permuted [Hepple 1990, Morrill 1992a, Moortgat and Oehrle 1993].

3. The Logic of Quantification

There have been two primary approaches to quantification within categorial grammar. The first was introduced by Montague [1970b], and involves a logical scheme of quantifying-in. We adopt this approach, using the refined version introduced by Moortgat [1988] for use with Lambek-style logical grammars. The second approach, developed by Hendriks [1987, 1990], and later in a polymorphic version by Emms [1990], is similar in flavor to Steedman's CCG approach to unbounded dependencies in that it involves the use of type-shifting. In particular, a family of types and categories are assigned to quantifiers to account for their distribution. In this way it is similar to the approach to quantification

[11] Both the simple categorial grammar with only application, and Lambek's categorial grammars, have both been shown to generate all and only the context-free languages [Bar-Hillel, Gaifman and Shamir 1960, Pentus 1992], even if restricted to only one variety of slash, either forward or backward.

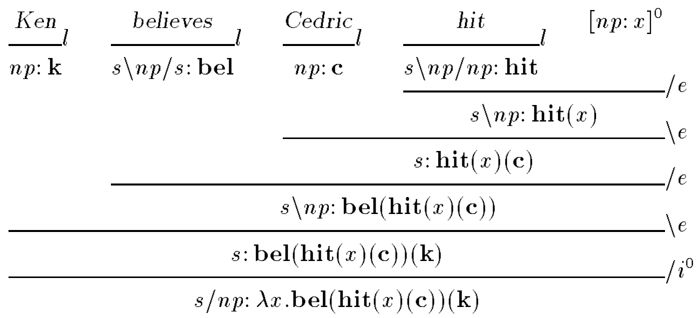

Figure 7: Slash Introduction Example

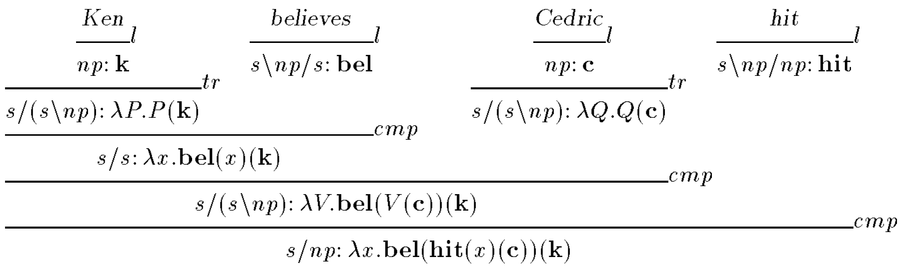

Figure 8: CCG Analysis with Composition and Type Raising

He also included dual versions of type raising and composition involving backward slashes. Using these schemes, along with the slash elimination schemes which are always assumed in categorial grammars, the derivation in Figure 7 can be carried out as in Figure 8. Steedman noted that analyses in this system were like those of phrase structure grammars, but could almost always be analyzed in a purely left branching fashion. Steedman also generalized the notion of composition beyond the simple case above, for instance to allow A/B and $B/C/D$ categories to be combined.

All of the instances of composition and type raising can be derived in Lambek's calculus. But the converse does not hold; not every instance of Lambek's schemes can be derived by simple type raising and a finite number of composition schemes. Instead, Zielonka [1981] showed that composition must be generalized in the manner of Geach [1972], and combined with the simple type raising schemes above, in order to generate a system that is weakly equivalent

$$\cfrac{\cfrac{\mathit{Brad}}{np\colon \mathbf{brad}}\,l \qquad \cfrac{\cfrac{\mathit{loves}}{s\backslash np/np\colon \mathbf{love}}\,l \quad \cfrac{\mathit{Pittsburgh}}{np\colon \mathbf{pgh}}\,l}{s\backslash np\colon \mathbf{love}(\mathbf{pgh})}\,/e}{s\colon \mathbf{love}(\mathbf{pgh})(\mathbf{brad})}\,\backslash e$$

Figure 6: Slash Elimination Example

Natural deduction proofs can be read as yielding sequents of the same sort as used in the sequent calculus; the sequence of assumptions forms the antecedent of the sequent and the root the consequent. For instance, the derivation in Figure 6 corresponds to the sequent

(14) *Brad*, *loves*, *Pittsburgh* $\Rightarrow s\colon \mathbf{love}(\mathbf{pgh})(\mathbf{brad})$

Note that the applications of the cut rule which would be required to derive this sequent using the sequent calculus are implicit in the natural deduction presentation.[10]

Things are a bit trickier in the case of the introduction schemes, often known as *rules of proof*, which correspond to the right rules in the sequent calculus. The introduction schemes allow hypothetical reasoning of the kind used in natural deduction approaches to implicational logic. For instance, to establish that $\phi \rightarrow \psi$ follows from a set of assumptions, we may assume ϕ as an additional assumption in the proof of ψ. Such an assumption is then eliminated after ψ is proved, thus establishing that $\phi \rightarrow \psi$ follows from the original assumptions. In Lambek's categorial logic, $/i$ allows us to derive $B/A\colon \lambda x.\alpha$ from a sequence of assumptions, if we can prove $B\colon \alpha$ from the same sequence of assumptions with the addition of $A\colon x$ as the rightmost assumption. Such a derivation is said to *discharge* the hypothetical assumption $A\colon x$, which is notated by surrounding the assumption in brackets, and indexing it with an integer to indicate at which step the assumption was discharged. An illustration of a derivation involving the introduction rule is given in Figure 7. This example shows how from the assumptions *Ken*, *believes*, *Cedric*, *hit*, and $np\colon x$, in that order, we can derive $s\colon \mathbf{believe}(\mathbf{hit}(x)(\mathbf{c}))(\mathbf{k})$. In the last step of the derivation, we discharge the assumption of $np\colon x$, abstracting over its variable x and over its argument np to produce the result $s/np\colon \lambda x.\mathbf{bel}(\mathbf{hit}(x)(\mathbf{c}))(\mathbf{k})$. This example also illustrates the unbounded nature of assumption introduction and discharge in Lambek's categorial grammars.

In Steedman's [1985, 1987, 1988] Combinatory Categorial Grammars, phrase structure schemes for type raising and composition are used to achieve effects similar to those of Lambek's scheme for slash introduction.

(15) a. $A\colon \alpha \Rightarrow A/(B\backslash A)\colon \lambda P.P(\alpha)$ [Forward Type Raising]

b. $A/B\colon \alpha \quad B/C\colon \beta \Rightarrow A/C\colon \lambda x.\alpha(\beta(x))$ [Forward Composition]

[10] See [Girard, Taylor and Lafont 1987] for more details on the correspondence between sequent calculi and natural deduction.

$$\cfrac{\cfrac{\cfrac{}{s\backslash np/np\colon \mathbf{hit},\ np\colon x \Rightarrow s\backslash np\colon \mathbf{hit}(x)}\,/l \qquad \cfrac{}{np\colon \mathbf{a},\ s\backslash np\colon \mathbf{hit}(x) \Rightarrow s\colon \mathbf{hit}(x)(\mathbf{a})}\,\backslash l}{np\colon \mathbf{a},\ s\backslash np/np\colon \mathbf{hit},\ np\colon x \Rightarrow s\colon \mathbf{hit}(x)(\mathbf{a})}\,c}{np\colon \mathbf{a} \quad s\backslash np/np\colon \mathbf{hit} \Rightarrow s/np\colon \lambda x.\mathbf{hit}(x)(\mathbf{a})}\,/r$$

Figure 4: Derivation of *Albert hit*

Lexical Entries

$$\cfrac{e}{A\colon \alpha}\,l$$

Slash Elimination

$$\cfrac{\begin{matrix}\vdots & & \vdots\\ A/B\colon \alpha & & B\colon \beta\end{matrix}}{A\colon \alpha(\beta)}\,/e \qquad \cfrac{\begin{matrix}\vdots & & \vdots\\ B\colon \beta & & A\backslash B\colon \alpha\end{matrix}}{A\colon \alpha(\beta)}\,\backslash e$$

Slash Introduction

$$\cfrac{\cfrac{\begin{matrix}\vdots & [A\colon x]^n\\ \vdots & \vdots\end{matrix}}{B\colon \alpha}}{B/A\colon \lambda x.\alpha}\,/i^n \qquad \cfrac{\cfrac{\begin{matrix}[A\colon x]^n & \vdots\\ \vdots & \vdots\end{matrix}}{B\colon \alpha}}{B\backslash A\colon \lambda x.\alpha}\,\backslash i^n$$

Figure 5: Natural Deduction Lambek Calculus

more practical, as noted by Barry et al. [1991]. We provide a natural deduction presentation of Lambek's logic in Figure 5. Some discussion concerning the interpretation of these schemes is in order. We read the lexical scheme as stating that we can derive an expression's lexical entry from it. The slash elimination schemes, often known as *rules of use*, are analogues of the left sequent schemes. For instance, forward elimination, $/e$, allows us to combine a derivation of $B\colon \beta$, the structure of which is represented by the vertical ellipses, with a derivation of $A\backslash B\colon \alpha$, the structure of which is also ellided, to produce a derivation of $A\colon \alpha(\beta)$. Backward elimination is analogous. The use of these schemes is illustrated in Figure 6. Derivations simply involving elimination and lexical schemes are analogous to phrase-structure trees. Here the logical, albeit incomplete, nature of phrase structure approaches to categorial grammar is clear; the tree structure corresponds to the structure of a logical derivation.

$$\dfrac{\dfrac{}{John \Rightarrow np\colon \mathbf{j}}l \quad \dfrac{\dfrac{}{runs \Rightarrow s\backslash np\colon \mathbf{run}}l \quad \dfrac{}{np\colon \mathbf{j} \quad s\backslash np\colon \mathbf{run} \Rightarrow s\colon \mathbf{run(j)}}\backslash l}{np\colon \mathbf{j} \quad runs \Rightarrow s\colon \mathbf{run(j)}}c}{John\ runs \Rightarrow s\colon \mathbf{run(j)}}c$$

Figure 3: Lexical Analysis of *John runs*

A simple analysis illustrating lexical insertion, cut and the left slash rules is provided in Figure 3. Note that the use of $\backslash l$ corresponds to functional application semantically. It is called a *left* rule because it eliminates an occurrence of a constructor from the left side of a sequent. Left rules are often called rules of use, as they indicate how a constructor is to be used in a proof. The cut rule is simply used to chain analysis steps; it states that if a subsequence Γ can be analyzed as $B\colon\beta$, then $B\colon\beta$ can be used in place of Γ in a derivation.[9]

The *right* rules characterize proofs with occurrences of a constructor on the right hand side of a sequent. Such rules are often called rules of proof, as they indicate how to derive a category with a particular top-level constructor. Consider the application of the right rule $/r$ in Figure 4. Without the right rules, Lambek's system reduces to the purely applicative categorial grammar, originally developed by Ajdukiewicz [1935], and extended to directional slashes by Bar-Hillel [1950, 1953]. The left rules for the slash constructor play a crucial role in our system for generating scopings in incomplete phrases, for sloppy anaphora, for coordination, and for generating alternations with operators such as adverbials and negation.

The sequent-based presentation of Lambek's categorial logic is both logically straightforward and of practical utility in proving meta-theorems. But for object-level derivations, a natural deduction version of Lambek's logic is

[Moortgat and Oehrle 1993] for details and linguistic applications.

[9] Lambek's original presentation of **L** was cut-free in that every provable sequent could be derived without the use of cut. To achieve this, the effect of cuts has to be built into the left rules, as in Gentzen's original presentation of sequent rules for propositional logic:

$$\dfrac{\Gamma_2 \Rightarrow B\colon\beta \qquad \Gamma_1,\ A\colon\alpha(\beta),\ \Gamma_3 \Rightarrow C\colon\gamma}{\Gamma_1,\ A/B\colon\alpha,\ \Gamma_2,\ \Gamma_3 \Rightarrow C\colon\gamma}\backslash l'$$

$$\dfrac{\Gamma_2 \Rightarrow B\colon\beta \qquad \Gamma_1,\ A\colon\alpha(\beta),\ \Gamma_3 \Rightarrow C\colon\gamma}{\Gamma_1,\ \Gamma_2,\ A\backslash B\colon\alpha,\ \Gamma_3 \Rightarrow C\colon\gamma}/l'$$

Lambek did not consider lexical entries, but those too may be integrated with the cut rule, as in:

$$\dfrac{\Gamma_1,\ A\colon\alpha,\ \Gamma_2 \Rightarrow B\colon\beta}{\Gamma_1,\ e,\ \Gamma_2 \Rightarrow B\colon\beta}l' \qquad [e \Rightarrow A\colon\alpha \in \mathsf{Lex}]$$

Note that all of these rules can be derived from our presentation. One benefit of Lambek's presentation is that it shows why derivability is decidable; the antecedents of sequents reduce their complexity, as measured by the sum of the number of expressions and slash constructors. We will not be concerned with cut-free presentations of our calculi, but the reader is referred to Moortgat and Oehrle [1993] for discussion.

Lexical Entries

$$\frac{}{e \Rightarrow A\colon \alpha}\,l \qquad [e \Rightarrow A\colon \alpha \in \mathsf{Lex}]$$

Slash Left

$$\frac{}{A/B\colon \alpha,\ B\colon \beta \Rightarrow A\colon \alpha(\beta)}\,/l \qquad \frac{}{B\colon \beta,\ A\backslash B\colon \alpha \Rightarrow A\colon \alpha(\beta)}\,\backslash l$$

Slash Right

$$\frac{\Gamma,\ A\colon x \Rightarrow B\colon \alpha}{\Gamma \Rightarrow B/A\colon \lambda x.\alpha}\,/r \qquad \frac{A\colon x,\ \Gamma \Rightarrow B\colon \alpha}{\Gamma \Rightarrow B\backslash A\colon \lambda x.\alpha}\,\backslash r$$

$$[\Gamma \text{ non-empty, } x \text{ fresh}]$$

Identity

$$\frac{}{A\colon \alpha \Rightarrow A\colon \alpha}\,i$$

Cut

$$\frac{\Gamma_2 \Rightarrow B\colon \beta \qquad \Gamma_1,\ B\colon \beta,\ \Gamma_3 \Rightarrow A\colon \alpha}{\Gamma_1,\ \Gamma_2,\ \Gamma_3 \Rightarrow A\colon \alpha}\,c$$

Figure 2: Sequent Presentation of Lambek's Associative Calculus

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Permutation	**Weakening**	**Contraction**
$\dfrac{\Gamma,\ \phi,\ \psi,\ \Gamma' \vdash \xi}{\Gamma,\ \psi,\ \phi,\ \Gamma' \vdash \xi}$	$\dfrac{\Gamma \vdash \psi}{\Gamma,\ \phi \vdash \psi}$	$\dfrac{\Gamma,\ \phi,\ \phi,\ \Gamma' \vdash \psi}{\Gamma,\ \phi,\ \Gamma' \vdash \psi}$

Logics without some or all of these rules are said to be *substructural*. The full analogy to logic is brought out by viewing the functor categories A/B and $A\backslash B$ as propositions of the form $B \rightarrow A$, with basic categories as propositional constants. In this way, our elimination rules are instances of modus ponens, and our introduction rules represent hypothetical reasoning.

The connection to terms is then given by the Curry-Howard morphism, which relates propositional implicational formulas and simple types, and also relates proofs in implicational logics to the construction of well-typed λ-terms. The morphism component of the correspondence stems from the normalization structure; normalizing proofs in intuitionistic implicational logic corresponds to η-reduction and β-reduction of λ-terms [Girard, Taylor, and Lafont 1987].

An even stricter substructural regime could be enforced by taking the antecedents in sequent to be bracketed strings. In this case, the result is Lambek's non-associative calculus. Adding a structural rule of association to the non-associative calculus results in the associative calculus.

One of the recent innovations in categorial grammar has been to allow structural modalities, such as the ! of linear logic, which allow the importation of the resource management of a more lenient logic, and modalities such as the modal logic **S4**'s $\Box$, which can be used to limit more lenient logics to stricter resource management. See [Hepple 1990a], [Morrill 1992a], and

Categorial grammar is completely lexicalist, assuming a universal deductive system, and relegating all language-specific information to the lexicon. In a categorial grammar, the lexicon provides a relation between expressions, syntactic categories and λ-terms of the appropriate type. Here, we assume that a lexicon is given by a finite[6] relation $\mathsf{Lex} \subseteq \mathsf{Exp} \times (\mathsf{Cat} \times \mathsf{Term})$, with the additional restriction that if $\langle e, \langle A, \alpha\rangle\rangle \in \mathsf{Lex}$, then α is a λ-term of the type of A. For notational convenience, we will write $e \Rightarrow A\colon \alpha$ if $\langle e, \langle A, \alpha\rangle\rangle \in \mathsf{Lex}$. This definition allows lexical ambiguities, where a given expression can be assigned to multiple categories and/or meanings. We do not discuss methods for structuring the categorial lexicon, but see [Carpenter 1992a] and [Pollard and Sag 1987].

Lambek's proof theory characterizes a relation between expressions and their meanings which is mediated by syntactic category assignments. The basic sequents in the Lambek calculus are of the form:

(13) $A_1\colon \alpha_1, \ldots, A_n\colon \alpha_n \Rightarrow A\colon \alpha$

A provable sequent of the above form is taken to indicate that expressions of categories A_i with meanings α_i can be concatenated to form an expression of category A with meaning α. In addition, a lexical insertion scheme connects expressions with their lexical entries, which consist of category-meaning pairs.[7] Lambek presented his categorial grammar in the form of a sequent calculus. We present Lambek's calculus, enriched with van Benthem's approach to semantic assignment, in Figure 2. We must be careful in this system to use fresh variables for the right rules; no two right rules in a derivation are allowed to introduce the same variable. Furthermore, all of our later rules will also be subject to this restriction. In addition, we require the variables introduced by the right rules to be of the appropriate type. A simple inductive argument suffices to show that if we use appropriately typed variables, then every occurrence of a λ-term in a derivation will be of the appropriate type for the category to which it is linked. All of our subsequent rule schemes will be subject to this same restriction, and we will not mention type soundness again. Finally, we require that the sequences Γ in the right rules for slashes be non-empty. Without this restriction, we would be able to derive unwanted categorizations for the empty string, such as $n/n\colon \lambda P.P$, from the combination of the right slash rule and the axiom instance $n\colon P \Rightarrow n\colon P$.

It is important to note that for sequents of the form $\Gamma \Rightarrow A\colon \alpha$, we take Γ to be a sequence of expressions and category-meaning pairs. We use the notation Γ_1, Γ_2 for the concatenation of sequences.[8]

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a set of basic expressions. This approach was the basis for Lambek's [1961] non-associative calculus. For a survey of the ramifications of the choice of expression algebra, both on the logic and its proof theory, see [Hepple 1990a], [Morrill 1992a] and [Moortgat and Oehrle 1993].

[6] See Carpenter [1991, 1992a] for a discussion of the linguistic utility and computational drawbacks of adopting lexical rules which potentially generate an infinite lexicon.

[7] The lexical insertion scheme was not part of Lambek's original presentation, nor is it used by many authors. Instead, the lexicon is usually related to the sequents as we have just outlined.

[8] In classical and intuitionistic implicational logics, the antecedents can also be interpreted as sequences, providing we allow the following structural rules:

Category	*Description*
n/n	adjective
np/n	determiner
$n\backslash n/np$	(nominal) preposition
$n\backslash n$	relative clause, (nominal) prepositional phrase
$s\backslash np$	verb phrase, intransitive verb
$s\backslash np/np$	transitive verb
$s\backslash np/(s\backslash np)$	adverb, auxiliary, intransitive control verb

Figure 1: Some Useful Categories

A category $A\backslash B$ or A/B is said to be a *functor* category and to have a *domain* or *argument* category of B and a *range* or *result* category of A.[4] A functional category of the form A/B is said to be a *forward functor* and looks for its B argument to the right, while the *backward functor* $A\backslash B$ looks for its argument to the left. Functor categories are associated with functional types in a straightforward way:

(12) $Typ(A/B) = Typ(A\backslash B) = Typ(B) \rightarrow Typ(A)$

With our choice BasCat of basic categories, some useful functional categories in Cat are given in Figure 1. We drop parentheses according to the convention that all of our category constructors are left associative, and that functional type constructors are right associative. Note that there are infinitely many categories, which is unproblematic for categorial logic, for the same reason that the infinity of propositional formulas is unproblematic for propositional logic, namely that categoryhood is easily decidable. In fact, our analyses exploit the open-ended nature of the category system in fundamental ways.

2.2. *Lambek's Associative Calculus*

To model expressions, we assume a finite set BasExp of basic expressions, and take the full set of expressions to be the collection of non-empty strings over the basic expressions, notated $\mathsf{Exp} = \mathsf{BasExp}^+$. We use $\cdot$ for the operation of concatenation, which we often omit, representing concatenation by juxtaposition. Concatenation obeys associativity in that $a \cdot (b \cdot c) = (a \cdot b) \cdot c$, so we will write both as $a \cdot b \cdot c$ or just $a\ b\ c$.[5]

[4] We employ Steedman's [1985] notation for categories, rather than Lambek's, because it allows logical types to be read off categories directly and because it allows more parentheses to be eliminated in our grammars. Lambek used the notation $B\backslash A$ for our $A\backslash B$.

[5] We have modeled expressions as elements of a free semigroup $\langle \mathsf{BasExp}^+, \cdot \rangle$ generated by concatenation over BasExp. It is possible to interpret categorial grammars in arbitrary groupoids $\langle \Sigma, \cdot \rangle$, where Σ is an arbitrary set of expressions structured by a binary operation $\cdot$. The most common such choice is free groupoids, which correspond to bracketed strings over

derivations on the other. This connection provides a strictly compositional semantic interpretation scheme. In this section, we present a proof-theoretical version of Lambek's categorial logic.

2.1. *Basic and Functor Categories*

As categorial grammar is designed to relate expressions to their meanings, we begin with a few comments about meanings. We represent meanings with λ-terms drawn from Church's simply typed λ-calculus.[2] For our purposes, two primitive semantic types suffice:

(9)

Type	*Domain*
Ind	Individuals
Prop	Propositions

We use $\sigma \rightarrow \tau$ for the type of functions from objects of type σ to objects of type τ. For each type τ, we assume a collection of constants, Con_τ, and a countably infinite set of variables, Var_τ. We assume that there are logical constants for the standard logical operators, quantification and equality, which respect the assignment of truth to propositions.[3]

In order to construct syntactic categories, we begin with a finite set BasCat of *basic categories*. Each basic category C is associated with a (not necessarily primitive) semantic type $Typ(C)$. For our purposes, it will suffice to assume the following basic categories and type assignments:

(10)

Category	*Type*	*Description*
np	Ind	noun phrase
n	Ind→Prop	noun
s	Prop	sentence

In addition to the basic categories, categorial logics provide a repertoire of category *constructors*, which are used to freely generate the full set Cat of categories from the set of basic categories BasCat. We begin with the binary "slash" constructors, introduced by Bar-Hillel [1950], following Ajdukiewicz [1935], which allow us to construct the set Cat of categories as the least set such that the following holds.

(11) a. $\mathsf{BasCat} \subseteq \mathsf{Cat}$

b. $A/B, A\backslash B \in \mathsf{Cat}$ if $A, B \in \mathsf{Cat}$

[2] Other authors have chosen different type systems in which to interpret categorial grammars. For instance, Chierchia and Turner [1988] interpret a Montagovian categorial grammar over the monotyped λ-calculus. But as we are more interested in the syntax/semantics interface, and not in particular semantic assumptions, our choice of type system is of no particular significance.

[3] Our purpose is not to present a theory of intensionality, and thus we will not be particularly concerned with the structure of the domain of propositions, other than that there be some method of extracting the truth or falsity of a proposition from it and that the standard logical operations are defined over it. For concreteness, propositions might be taken to be functions from possible worlds to truth values, empiricist criteria of verification, as structured propositions of some kind, or as unanalyzed elements of a Heyting algebra partitioned into true and false propositions.

We also account for the interaction between quantification and anaphora.

(7) a. Every Englishman supports his football team.
b. Only John believes he will win the upcoming election.

In (7)a, we get two readings. One of these readings allows the possessive pronoun *his* to be read deictically or as picking up a previously introduced antecedent. The second picks up the subject of the sentence, *every Englishman*. Following Montague, we treat pronouns as variables, which allows them to pick up arbitrary antecedents. The two readings of (7)a then follow from the nature of quantification. The case of (7)b is more interesting, as it involves a a distinction in the scope of the pronominal binding. We treat it as a strict/sloppy distinction, where either John is the only one who believes John will win, or John is the only one with a faith in his or her own ability to win. Again, we need no additional stipulations; the possibilities are predicted by the combination of our approach to anaphora and Lambek's categorial logic. The same mechanism also allows sloppy readings of anaphors in verb phrase ellipsis without recourse to reconstructive analyses, even of the logical variety proposed by Dalrymple, Shieber and Pereira [1991].

In the interest of space, we leave aside the independent issue of the interpretation of plural noun phrases. In other sources [Carpenter 1992b], we address this issue, noting that the logical approach to quantifiers solves some difficult puzzles such as those arising from the following examples.

(8) a. Three examiners marked six scripts.
b. The students gathered outside the deans office and shouted in protest.
c. The committees met.
d. John and Bill believe they like each other.

For instance, our approach to plurals allows for the eight readings of (8)a, predicted by Davies [1989] from the scope alternations and collective/distributive distinction, as well as his ninth reading in which there are three examiners and six scripts and each examiner marked each script. In cases such as (8)b, the categorial logic of Lambek combined with a natural approach to plurals accounts for the possibility of truthfully coordinating "distributive" and "collective" predicates. We also get a natural account of the ambiguity in (8)c, and of the multiple ambiguities arising from plural anaphora, reciprocals and scope in (8)d.

2. Categorial Logic

Our point of departure is Lambek's [1958] associative calculus for categorial grammar. In the generative tradition, Lambek provides a decidable proof theory which specifies the relation between expressions and their syntactic categories. As noted by van Benthem [1983b], the Curry-Howard morphism [Howard 1980], about which we have more to say below, provides the natural bridge between categories and their types on the one hand, and between syntactic and semantic

In (5)a, we see how quantifiers, as boolean categories, are allowed to be conjoined without introducing scope ambiguities. This is simply an instance of the general boolean coordination scheme. The introduction scheme for Moortgat's quantification constructor also predicts the coordinability of quantifiers and names, which we treat as purely referential, as seen in examples such as (5)b. In (5)c, we see how determiners behave when their restriction is a coordinated noun. In the case of (5)c, there is an ambiguity between two readings, one in which everyone who is both a vegetarian and socialist demonstrated, and a second in which every vegetarian demonstrated and every socialist demonstrated. We see a similar ambiguity in (5)d, where the scope of the quantifiers is coordinated. On one reading, every student ran or every student jumped, and on the other, every student either ran or jumped. Partee and Rooth's [1983] example, (5)e, with an intensional verb taking a coordinated quantified object, leads to a three-way ambiguity. On the narrow scope coordinator, de dicto reading, John is seeking anything that happens to be either a pen or a pencil. On the de re reading, there is a particular pen or a particular pencil such that John is looking for it. The third reading involves wide-scope for the coordinator, but a de dicto reading for the objects, in which John is either seeking a pen, and he doesn't care which pen, or he's seeking a pencil and he doesn't care which pencil. Partee and Rooth make this third reading more plausible by supposing it is followed with *but I don't know which*. These three readings fall out from the combination of hypothetical reasoning in Lambek's calculus and the standard approach to de dicto/de re ambiguities, in which intensional verbs take generalized quantifiers as arguments. The same mechanism accounts for the possibility of the coordination of intensional and non-intensional verbs, as in (5)f. Our grammar even captures the fact that in such cases, the intensional verb can be read de dicto, and the extensional verb de re. In (5)g, we see that coordination of material involving a quantifier does not allow quantifiers in the coordinated material to interact with each other. On the other hand, coordinated quantifiers can take wide scope with respect to other constituents, as seen in (5)h, where a different student could have read each book and interacted with each presentation. All of these facts can be derived using Lambek's introduction schemes for the categorial complement constructors.

Quantification also has non-trivial interactions with unbounded dependency constructions.

(6) a. Every person that someone took a picture of was pleased.
 b. A table every leg of which broke fell over

In (6)a, we see that the scope of the embedded quantifier must remain narrow. Our account of this fact follows from the logical structure of our derivations. In this way, our analysis provides a categorial analogue of Pereira's [1991] deductive system. In (6)b, which involves pied-piping of a quantified noun phrase, we see that the quantifier must take scope within the relative clause, and cannot escape to take sentential scope, which would involve a different table for each leg. Somewhat surprisingly, Morrill [1992b] has shown that relative pronouns can be treated as a kind of quantifier, providing a general account of their uses in pied-piping and in non-pied-piping cases.

Such examples are problematic for accounts of quantification based on storage mechanisms, and also for transformational systems in which subjects are VP-external. In (3)a, there is an ambiguity between wide and narrow scope of the subject quantifier with respect to the negation operator. In the next example, (3)b, the quantifier is allowed to scope either wholly outside, wholly inside, or in between the adverbial and the negation. These cases receive a categorial analysis similar to that proposed for raising control in (3)c and (3)d, both of which admit scope ambiguity. Next, we see how quantifiers appearing as equi-type controllers fail to provide narrow scope alternatives, as in (3)e, which is unambiguous. This is due to a lexical difference between raising and equi controllers, in which equi verb controllers are restricted to noun phrases, rather than allowing generalizing quantifiers. In the last example, (3)f, there is an ambiguity based on whether or not the quantifier's scope is within the verb phrase complement or over the entire sentence. To account for these facts, we follow the standard categorial approach to control [Steedman 1985]. The interactions stem from the ability of Lambek's categorial calculus to introduce and discharge hypothetical categories. The distinction between these two forms of control is treated lexically.

Scope ambiguities similar to those for explicit operators occur with respect to the scope of tenses, relative to both nominal quantifiers, nominal tenses, temporal adverbs, and quantificational adverbs. Consider the following examples.

(4) a. Every kid ran.
    b. A student would frequently interrupt class to ask a question.

Even in simple sentences such as (4)a, there is ambiguity concerning whether there was a single time in the past at which every kid ran, or whether for every kid there is simply some time in the past at which the kid ran. Generalized quantifiers can take scope within or outside of temporal adverbials, as seen in (4)b. We will not consider the interactions of tense and scope here, but refer the reader to [Hinrichs 1988], both for further data on the interactions between tense and quantification, and for a Reichenbachian approach to tense which could be integrated with our analysis of nominal quantifiers.

The interaction between quantifiers and coordination is often problematic for theories with otherwise adequate, independent accounts of the two phenomena. Consider the following cases.

(5) a. One professor and every student attended the lecture.
    b. Jones worships Smith and every other English professor.
    c. Every vegetarian and socialist demonstrated on the cut.
    d. Every student ran or jumped.
    e. John is looking for a pen or a pencil.
    f. John was looking for and finally found a pen.
    g. Every student liked but some professor disliked the talk.
    h. At least one student read every book and interacted with every multimedia presentation.

within relative clauses, and within possessive determiners.

(2) a. A student in every class failed.
b. A picture of every student was taken.
c. A student who every kid likes met with Fred.
d. What every kid rode was a bicycle.
e. Every kid's favorite toy broke.
f. The receptionist for every office is underpaid.

The first example, (2)a, displays a two-way ambiguity. One reading involves a student who is in every class; the other involves a possibly distinct student for each class. This ambiguity must be captured by allowing quantifiers to reduce within nominal complements, both to treat (2)a and (2)b uniformly, and to prevent the well known problem arising from unbound variables when the embedded quantifier is allowed sentential scope that is narrower than the embedding quantifier. Unbound variables are eliminated by the logical structure of our deductive rule for quantifier insertion, rather than by the ad hoc stipulation of May [1985], Hobbs and Shieber [1987] and Pollard and Sag [in press], or by complicating semantic representations, as in nested accounts of Cooper storage [Keller 1988; Gerdemann and Hinrichs 1991]. Next, it has been generally assumed that quantifiers are not allowed to escape from relative clauses to take sentential scope, thus yielding only one reading for (2)c. Such restrictions are easily captured using Morrill's [1990a, 1992b] approach to islandhood in categorial logic; in fact, such an approach also captures the islandhood of relative clauses with respect to extraction. Example (2)d shows that quantifiers can also be bound within free relative clauses, but can also escape to take sentential scope. On the relative internal reading, there would be a single object that every kid rode, which happens to be a bicycle. Under the sententially scoped interpretation, each kid could have ridden a different bicycle. Finally, when a quantifier occurs as a possessor, as in (2)e, we see that there is an ambiguity depending on whether the definite existential introduced by the possessive takes scope narrowly or widely with respect to the possessor. In this case, there can be a toy which is every kid's favorite, or a possibly different favorite toy for each kid. Both possessives and free relatives will be treated as a kind of definite description operator, from which the scoping facts follow. Our referential approach to definites also accounts for the ambiguity of (2)f, which admits a reading in which there is a different receptionist for each office.

There are subtle interactions between quantifiers and other operators, such as control verbs, adverbials and negative particles.

(3) a. Everyone didn't attend the party.
b. Everyone probably didn't attend the party.
c. Someone seems to be in attendance.
d. John believes everyone to have read a book.
e. John persuaded everyone to be quiet.
f. John wants to like everyone.

same fault as other non-logical models, namely that they require filters such as variable binding conditions to reject otherwise well-formed derivations.

Many generalizations concerning the logical possibilities for the behavior of natural language quantifiers and determiners have been recognized and catalogued [Barwise and Cooper 1981; Keenan and Stavi 1986; van Benthem 1983a, 1984]. Our approach, based on generalized quantifiers and determiners, is of course compatible with these empirical observations. But our primary concern is the syntax/semantics interface and the ability of quantifiers to take alternative scopes, rather than the independent characterization of possible natural language determiners and quantifiers.

The remainder of this paper is organized as follows. In the rest of this introductory section, we illustrate the kinds of data with which we will be concerned. We begin by introducing introduce Lambek's [1958] categorial logic, which deals exclusively with syntactic phenomena, along with van Benthem's [1983b] method, derived from the Curry-Howard morphism [Howard 1980], of deriving logical representations from syntactic representations.

The most obvious occurrences of quantified noun phrases are as arguments to verbs, as in the following examples.

(1) a. A student read every book.
    b. John gave a book to every child.
    c. John is seeking a good job.

Example (1)a is the ordinary case of a subject/object scope ambiguity; the sentence can mean either that for every student, there was some book that student read, or that there was a single student who read every book.[1] Our approach to quantification is basically that of Moortgat's [1991] categorial logic formulation of Montague's term insertion rule [1970b]. The basic principle is that a quantifier acts as a simple noun phrase and then takes scope at some sentence in which the quantifier is embedded. One advantage of Moortgat's formulation over Montague's is that there is no need for dummy expressions such as he_n. The next example, (1)b, also has two readings, depending on the relative scope of the object and prepositionally marked object. The final example, (1)c, involves an intensional argument position, which gives rise to the so-called de dicto/de re ambiguity. On the de re reading, John is pursing a particular job, whereas on the de dicto reading, his search is for any old job, as long as it is a good one. Again following Montague, we simply allow the intensional verbs like *seek* to take generalized quantifiers as arguments. The introduction rule for quantifiers, which is the dual of the term insertion rule for eliminating quantifiers, does the rest of the work in the same way as Montague's type raising.

In addition to simple cases of quantifiers occurring as verbal complements, we consider their role within the noun phrase. In particular, we consider quantifiers serving as nominal prepositional objects, as the objects of relational nouns,

[1] We assume, following Fodor and Sag [1982], that the indefinite article a can function as a generalized quantifier; we will have little to say concerning its ability to introduce a discourse referent.

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QUANTIFICATION AND SCOPING: A DEDUCTIVE ACCOUNT

ABSTRACT. In this paper, we argue that the grammatical scopings of quantifiers should be treated by deductive methods. In support of this position, we offer a logical treatment of almost all previously proposed substantive constraints on quantifier scoping, including those imposed by coordinate structure, control verbs, unbounded dependency constructions, anaphoric dependency and nested dependent quantifiers. These are correctly captured by a handful of linguistically motivated and logically natural inference schemes for quantification, coordination and unbounded dependency, combined with the previously motivated function introduction and elimination schemes of categorial logic. In addition, we argue that phrase-structure and transformational accounts of similar phenomena at best provide an approximation of the logical approach.

1. Introduction

In this paper, we argue that an adequate approach to natural language quantification, and in particular, its interaction with other mechanisms such as coordination, control and unbounded dependencies, can best be formulated as a deductive system using a categorial logic. The logical perspective on natural language syntax and semantics is enlightening from both an empirical and a theoretical standpoint. Our goal is to show that a simple categorial logic, amended with inference rules for quantification, unbounded dependencies and coordination, provides an elegant and precise solution to the often puzzling behavior of quantifiers and their interaction with other grammatical phenomena.

A secondary goal of this paper is to show that previous accounts of quantification, especially accounts of its interaction with other constructions, can at best be viewed as approximations to the logical account provided here. In particular, phrase structure based accounts, such as Cooper's [1982, 1983] formulation of quantifier storage and its later nested formulation [Keller 1988; Gerdemann and Hinrichs 1990], the GPSG slash-based account of unbounded dependencies [Gazdar 1981b], which is closely related to the phrase structure based catgorial accounts [Geach 1972; Steedman 1985], can both be viewed as approximating the natural deduction version of our inference rules. The insight behind our logical approach is that the interactions of various scoping operations, such as quantification and unbounded dependencies, follows from their characterization as inference rules, rather than from ad hoc constraints placed on representations. Similarly, transformational accounts of quantifier scoping [May 1985], to the extent that they are correct, can be seen as nothing more than "phrase marker" approximations of logical derivations. Insofar as they freely generate scope ambiguities using general transformations, they suffer the