Interpreting similarity using attribute spaces and generalized measure functions

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Similarity in natural language

- Similarity is a core concept in human cognition,
  e.g., in classifying objects / situations

- Natural languages provide multiple ways to express similarity,
  e.g, adjectives *ein ähnliches Auto* / *a similar car*
  demonstratives *so ein Auto* / *such a car, a car like this*

➔ Spell out the semantics of similarity expressions
accounting for the results on similarity in Cognitive Science and AI
without abandoning truth-conditional semantics

The basic idea

- Multi-dimensional attribute spaces:
  feature structures + additional structure

- Generalized measure functions:
  many dimensional counterparts of one-dimensional measure functions in degree semantics (Kennedy 1999)

- Generalized measure functions map individuals to points in multi-dimensional attribute spaces

- Similarity is spelt out as indistinguishability with respects to a given set of attributes.

Similarity demonstratives

Demonstratives expressing similarity (instead of identity):

German	*so, wie dies*
English	*such, like this*
Turkish	*böyle*
etc.	

(1) (speaker pointing to a car): *So ein Auto hat Anna.*
'Anna has a car like this.'

- Demonstration target: the car the speaker points at
- Referent of the NP: instance of the similarity class generated by the target,
  ≈ ad hoc kind

Semantics of ad-nominal *so*

(1) (speaker pointing to a car): *So ein Auto hat Anna.*
'Anna has a car like this.'

Similarity is a 3-place relation $\mathrm{SIM}(x, x_{target}, \mathcal{F})$

x NP referent

x_{target} target of pointing

$\mathcal{F}$ representation, including a set of dimensions of comparison

$[_{NP}\ [_{DET}$ *so ein*] *Auto*] ('such a car')

$[\![$*so*$]\!] = \lambda D.\ \lambda P.\ D(\lambda x.\ \mathrm{SIM}(x, x_{target}, \mathcal{F})\ \&\ P(x))$

$[\![$so ein$]\!] = \lambda P.\ \lambda Q.\ \exists x.\ \mathrm{SIM}\ (x, x_{target}, \mathcal{F})\ \&\ P(x)\ \&\ Q(x)$

$[\![$ *so ein Auto*$]\!] = \lambda Q.\ \exists x.\ \mathrm{SIM}(x, x_{target}, \mathcal{F})\ \&\ car(x)\ \&\ Q(x)$

Semantics of ad-adjectival *so*

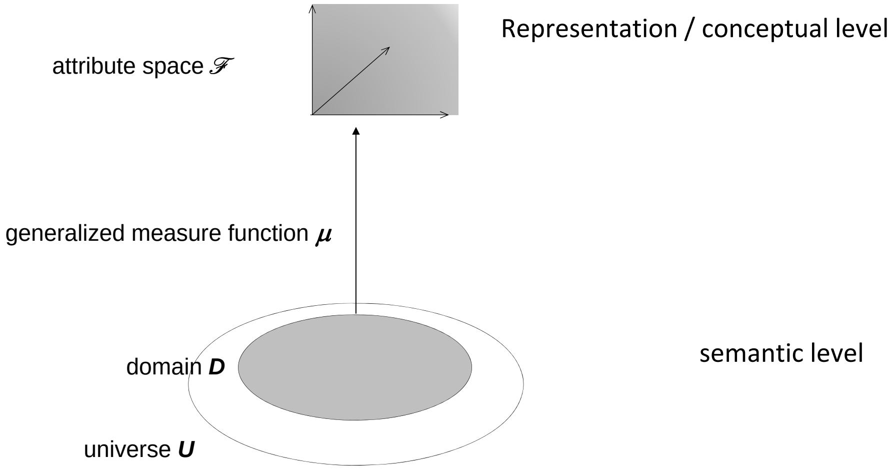
(2) (speaker pointing at a person): *So groß ist Anna.*
'Anna is that tall.'

- nominals: multiple dimensions of comparison
  selected by the context, restricted by the noun, nominal
  dimensions may relate to ratio / ordinal / nominal scales
- adjectives: one dimension of comparison
  given by the adjective's meaning
  dimension has a ratio scales
- [*so groß*] ('this tall')
  $[\![$*so*$]\!] = \lambda f.\ \lambda x.\ \mathrm{SIM}(x, x_{target}, \mathcal{F}(f))$
  $[\![$*so groß*$]\!] = \lambda x.\ \mathrm{SIM}(x, x_{target}, \mathcal{F}(height))$

Generalized measure functions

- Measure function associated with *tall* (Kennedy 1999):
  $\mu_{height}:\ D \rightarrow \Re$
- Suppose, relevant dimensions of comparison are
  DRIVE_TYPE: {diesel, gasoline, natural gas, electric}
  HORSEPOWER: $\Re^+$
  DOORS: {1 …5}
  EQUIPMENT: $\wp${rear assistance, lane guide, park pilot, BLIS}
  IMMOBILIZER: {0, 1}
- Generalized measure function associated with *car* (in the context):
  $\mu_{CAR}:\ D \rightarrow \text{DRIVE-TYPE} \times \text{HP} \times \text{DOORS} \times \text{EQUIPMENT} \times \text{IMMOBILIZER}$
  $\mu_{CAR}(x) = <\mu_{DRIVE\text{-}TYPE}(x), \mu_{HP}(x), \mu_{DOORS}(x), \ldots >$

Interim result – half of the picture



Representation / conceptual level

attribute space $\mathscr{F}$

generalized measure function $\boldsymbol{\mu}$

domain ***D***

semantic level

universe ***U***

How to implement similarity in $\mathcal{F}$?

Classification functions

- Multi-dimensional attribute spaces are given by a representation $\mathcal{F}$ specifying dimensions and corresponding measure functions and a set ***P**** of classification functions.
- Classification functions approximate natural language predicates on a conceptual level yielding corresponding truth values.

  $high\text{-}powered^*(\mu_{HP}(x))$ iff $high\text{-}powered\ (x)$ (for relevant x)

- Classification functions are defined, e.g., by a set of basic membership functions (crisp or fuzzy). The basic classification functions determine the maximal granularity.
- New classification functions are constructed from basis ones by conjunction, disjunction, negation and closure operators.

Similarity as indistinguishability

- Similarity is defined making use of classification functions such that two individuals are similar (with respect to given $\mathcal{F}$ and ***P****) iff the classification functions yield the same result when applied to corresponding points in $\mathcal{F}$:

  (*) SIM$(x, y, \mathcal{F})$ iff $\forall\ p^* \in P^*$: $p^*(\mu_F(x)) = p^*(\mu_F(y))$

- The similarity relation in (*) corresponds to the notion of indistinguishability in rough set theory (Pawlak 1998).
- The similarity relation in (*) is an equivalence relation.

Integrate attribute spaces into truth-conditional semantics

- Recall:
  Classification functions approximate natural language predicates on a conceptual level yielding corresponding truth values.

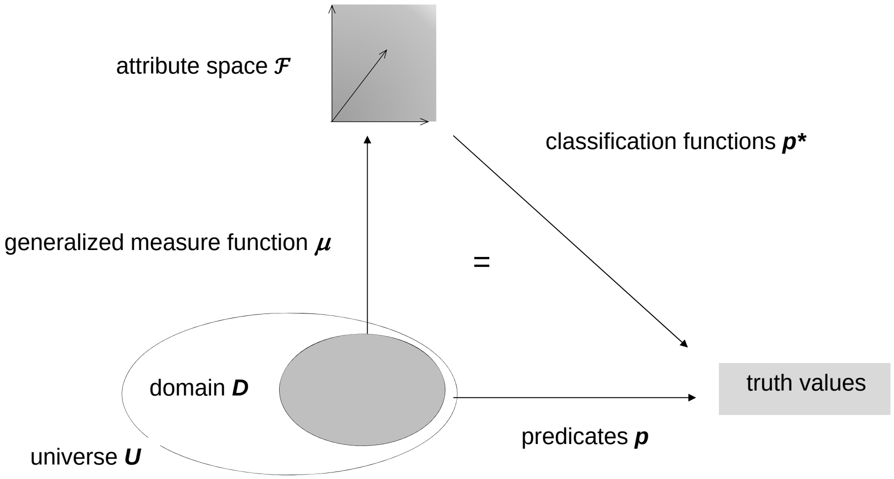
  $high\text{-}powered^*(\mu_{HP}(x))$ iff $high\text{-}powered\ (x)$ (for relevant x)

- Classification functions connect descriptions on the conceptual level to regular predicates:

  (**) $\forall p^* \in P^*$: $p^*(\mu_{\mathcal{F}}(x)) = p(x)$

- Classification functions warrant the integration of attribute spaces into truth-conditional semantics.

The full picture



attribute space $\mathcal{F}$

classification functions ***p****

generalized measure function $\boldsymbol{\mu}$

=

domain ***D***

truth values

predicates ***p***

universe ***U***

Domains

A ***domain*** is a quadruple $\mathcal{D} = < D, .^+, .^-, P >$ with:

- D a set
- $P = \{p_1, \ldots, p_n\}$ a set of predicates over D
- $.^+ : \{p_1, \ldots, p_n\} \rightarrow \wp(D)$ (positive examples)
- $.^- : \{p_1, \ldots, p_n\} \rightarrow \wp(D)$ (negative examples)
- $p_i^+, p_i^- \subseteq D$
- $p_i^+ \cap p_i^- = \emptyset$ (consistency)

Representations

A ***representation*** $\mathcal{F} = < F, \mu, .^*, \mathcal{D} >$ of a domain $\mathcal{D} = <D, .^+, .^-, P>$ is given by

- an **attribute space *F***
- a **measure function**
  $\mu : D \rightarrow F$ with $\mu(p_i^+) \cap \mu(p_i^-) = \emptyset$
- **classification functions**
  $.^* : P \rightarrow \Omega^F$ we call p^* an ***approximation*** of $p \in P$
- $p^*(\mu(p_i^+)) = \{\text{true}\}$ (consistency)
  $p^*(\mu(p_i^-)) = \{\text{false}\}$

Similarity

Similar as **undiscernable**

- $\mu(x) \sim_{P*} \mu(y)$ iff $\forall p^* \in P^*: p^*(\mu(x)) \leftrightarrow p^*(\mu(y))$

We get an order on the P^*s:

- $P^* \leq P^{*\prime}$ iff $\mu(x) \sim_{P*} \mu(y) \rightarrow \mu(x) \sim_{P*\prime} \mu(y)$

Given a domain $\mathcal{D} = < D, .^+, .^-, P >$ with representation $\mathcal{F} = < F, \mu, .^*, \mathcal{D} >$

- sim(x, y, $\mathcal{F}$) iff $\mu(x) \sim_{P*} \mu(y)$

Similarity as equivalence relation ?

Tversky (1977) argued against a metrical notion of similarity / distance

- triangle inequality is hardly compelling
- minimality is problematic
- symmetry is apparently false.

Transitivity: fixed dimensions of comparison
SIM(x, y, $\mathcal{F}$) & SIM(y, z, $\mathcal{F}$) → SIM(x, z, $\mathcal{F}$)

Reflexivity: required when interpreting the demonstrative *so* / *such*,
blocked when interpreting the adjective *ähnlich* / *similar*,

Symmetry: Gleitman et al (1996) show that Tversky's results are due
to figure-ground effects.

Graded similarity

While *so* / *such* are not gradable, *ähnlich* / *similar* are gradable

(3) a. *Anna hat ein ähnlicheres Kleid.*
Anna has a more similar dress.

b. * *Anna hat ein Kleid, das mehr so ist.*
* *Anna has a more such dress.*

Gradability of similarity:
x is more similar to z that y to z
iff there is a finer-grained classification system such that
x is similar to z but y is not similar to z

(***) more-sim$(x, y, z, \mathcal{F})$ iff $\exists \mathcal{F}'$ sim $(x, z\ \mathcal{F}')$ & $\neg$sim $(y, z, \mathcal{F}')$

where $\mathcal{F}'$ differs from $\mathcal{F}$ only in $P^* \leq P^{*\prime}$

Conclusion

- Natural languages provide multiple ways to express similarity,
  e.g, demonstratives (*so* / *such*) and adjectives (*ähnlich* / *similar*)
- the demonstratives *so* / *such* generate similarity classes induced by the demonstration target, which serve as ad hoc kinds
- Similarity is spelt out using
  - multi-dimensional attribute spaces, and
  - generalized measure functions.
- Similarity is defined as indistinguishability with respects to a given set of attributes, where
  - *so* / *such* require reflexivity, and
  - *ähnlich* / *similar* require irreflexivity

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