Towards a Category of Frames

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In my talk, I discuss how to represent frames in category theory. Categories are mathematical structures that allow us to model other structures from an abstract point of view (Mac Lane 1998). Basically, categories are given by a class of objects and by the morphisms between all pairs of objects of that class. If a morphism does not uniquely determine its pair of objects, we speak of a *pre-category*.

For the special case of feature structures, Moshier (1995) discusses so-called *domains* in category theory and proposes a model of information encoding with domains. The domains are based on the tree structure of feature structures. In contrast, frames, as defined in (Petersen 2007), are a generalization of feature structures because they can have an arbitrary graph structure. I will discuss in how far Moshier's considerations are applicable to frames.

To represent frames in category theory, I employ several interdependent kinds of categories. To define a frame, an underlying universe of discourse is needed, consisting of entities and their attributes. The values of these attributes are taken to be entities in the universe of discourse, too. A frame itself can be regarded as formed by its graph, that is a set of nodes and edges, plus the information it carries. On a frame, the information is given by labels which assign each node a type and each edge an attribute. Types and attributes are given by a type signature which consists of a hierarchy of types with the information of domain and range for each attribute.

As a basis, the universe of discourse can be described in a pre-category U, with the objects of the universe the objects of the category and the attributes in the universe, connecting objects and the values of their attributes, the morphisms of the category. Types can be regarded as a subclass of attributes, relating objects to themselves. Thus, if we define types this way, types cannot be distinguished from reflexive attributes.

A formal frame can be represented by a category G for its frame structure (that is its graph). Here, the nodes are the objects of the category and the paths (i.e. the edges and their concatenations) are the morphisms. On the other hand, the content of a frame is represented by a pre-category, F. Here, objects are sets of things (that is, sets of the objects in U). As in U, morphisms are given by attributes and types, where types are denoted by identity morphisms. F is just a pre-category, as the same attribute and the same type can occur in a frame more than once and thus the uniqueness requirement for morphisms fails. This makes it straightforward to define a functor from U to F that respects attributes and types and thus gives a well-formedness constraint for frame categories. On the other hand, F is closely connected to G, as types can be interpreted as labels for paths of length zero, again, being just special cases of attributes. Thus for each frame we have a functor connecting its frame pre-category to its graph category to express adequacy constraints. This functor allows to transfer other some constraints, as, for example a weak form of uniqueness constraint that is defined on F: Attributes are functional, so if a morphism goes from A to B and from A to C, we can conclude that $B=C$. This constraint cannot be defined directly on the category of the graph as the category of a graph is a proper category and thus its morphisms are uniquely assigned.

As in the examples already given, functors prove a useful tool to define constraints on frames. Apart from well-formedness constraints, there are constraints on values of frames, as Barsalou (1992) proposes them, e.g. monotonicity constraints. I discuss how to define such constraints in terms of categories.

Functors between categories for different frames can model relations between and operations on frames. For example, I discuss how frame-subsumption or composition can be defined via functors. As a next step, I regard the category *S* of the space of frames. Here, the objects are the frame categories and the morphisms are the functors between them. Some types of frames, like frames for lexicalized concepts or frames for relational concepts, can be captured as subcategories of the space of frames.

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