A Frame-Theoretic Primer on Reduction in Science

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The concept of reduction is central to the philosophy of science and to science itself. Intuitively speaking, a branch of science or a scientific theory is said to reduce to another branch of science or scientific theory when the ideas, ontology, laws and predictions of one are accounted in some way by the other. In this talk, we provide a frame-theoretic account of the concept of reduction. We first reconstruct the classical conception of reduction in frame-theoretic terms. This renders reduction as a sort of mapping relation between frames. We then consider a number of objections raised against the classical conception as well as proposed solutions. It is argued that although some progress has been made with neo-classical conceptions of reduction, various problems remain unsolved. We end the paper by proposing further modifications to the neo-classical accounts, modeling such modifications in frame-theoretic terms.

The classical conception of reduction goes back to Nagel (1961). According to this conception, a theory T reduces to a theory T' if, and only if, two conditions are met: (i) connectability: for every term F in T, there is a term G that is constructible in T' such that for any object a, Fa if, and only if, Ga and (ii) derivability: T is derivable from T', potentially bridge laws B and potentially restrictive conditions A. Nagel identified two types of reduction: homogenous and heterogeneous. In *homogeneous* reductions the reduced theory's vocabulary is either included in, or at least can be defined in terms of, the reducing theory's vocabulary. A frequently cited example is the reduction of Galileo's law of free fall to Newtonian physics. Since the former assumes that acceleration is constant at or near the Earth's surface while the latter takes it to be proportional to the force acting on the given body, a restrictive condition is required for the derivation. This takes the form of the constant g, which denotes the 'average' acceleration imparted on objects with small mass by the Earth's local gravitational field. *Heterogeneous* reductions require bridge laws to meet the connectability condition. Bridge laws connect the vocabulary of the reduced and reducing theories so that derivability can be achieved. In other words, bridge laws come into play only in heterogeneous reductions. A frequently cited example is the reduction of the second law of thermodynamics to statistical mechanics. The required bridge law connects temperature (a concept in thermodynamics) with mean kinetic energy (a concept in statistical mechanics).

A number of objections have been raised against the classical conception of reduction. One concerns the derivability requirement. Feyerabend (1962) points out that in the great majority of cases in actual science this requirement cannot be met because the reduced theory and the reducing theory are inconsistent. Another problem with the classical conception concerns the variance of meaning across theories. Thus although the concept mass appears in both classical mechanics and the special theory of relativity they do not mean exactly the same thing. In the latter mass is not an invariant quantity but increases as the velocity of an object nears that of the velocity of light.

Solutions to these and other objections are discussed in various places. Dizadji-Bahmani et al. (2010) argue that a neo-classical account of reduction modeled on Schaffner (see, for example, his 1967) offers the best hope to address most of these objections. Schaffner's innovation is to point out that what gets reduced is not the original theory T but a corrected version T^* that is strongly analogous to T. Thus, contra

Feyerabend, it can be argued that derivability is maintained, even though what gets derived from *T′* is *T**, not *T*. Two crucial modifications to Schaffner's model that Dizadji-Bahmani et al. make are that: (1) it is not necessary that every term of the reduced theory be connected to a term of the reducing theory and (2) it is not necessary that a term of the reduced theory be connected to exactly one term of the reducing theory. The first allows the modeling of partial reductions, while the second allows the modeling of multiple realisability.

We support the liberalisation of the notion of reduction carried out by Dizadji-Bahmani et al. and proceed to liberalise the notion further. We first motivate the move from *T* to *T** by arguing that it is natural for *T* and *T′* to be inconsistent in cases of progress. After all, a successor theory *T′* which allows more precise calculations of a given quantity necessarily conflicts with the calculations of its predecessor *T*. We then argue that this inconsistency can be produced by differences in the reduced and reducing theories beyond those mentioned in (1) and (2). For example, we posit that it is not even necessary that every term of *T′* be connected to a term of *T* or *T** since *T′* may have additional terms in its vocabulary. A case in point is the so-called Lorentz term γ in the special theory of relativity's conception of momentum. Although the classical conception of momentum and the relativistic one are both defined as a function of mass and velocity, the latter (which, by the way, is the reducing theory) states that it is also a function of γ. We also argue that the notion of 'being strongly analogous' needs to be put on a firmer footing. For example, given that some reductions turn out to be partial, in such cases *T** needs to be strongly analogous to only part of *T*. We model this and other proposed modifications to the classical conception of reduction in terms of mappings between frames. To give a rough account, reduction can be achieved so long as: (a) there is a bijective mapping between part of the structure of the frame of *T′* and the whole structure of the frame of *T** and (b) there is a transformation function from the frame of *T* to the frame of *T** that keeps at least some of the defining features of the former intact (e.g. by preserving some of its value-value, attribute-attribute or value-attribute constraints).

Dizadji-Bahmani, F. et al. (2010). Who's afraid of Nagelian reduction? *Erkenntnis* 73, 393-412.

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