# Analogical Concept Formation in Scientific Explanations

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# Outline

We are familiar with comparisons like:

- Electric current in a conductor is like water in a pipe
- Memes are like genes
- Interaction with God is like triangulation

⋮

Such comparisons are often seen as some kind of concept formation by analogies.

But what does it mean to form/construct concepts by analogies and what are they good for?

Overview:

- Part 1: Concept formation by analogies
- Part 2: Reductionism

# ANALOGIES

# Relevance of Analogies

Analogies are no hot-topic in philosophy of science.

To get nevertheless involved into recent debates I will try to embed results about analogies into modern theoretical frames:

- Conclusions by analogy: frame of confirmation theory (cf. [Hes64])
- Concept formation by analogy: frame of reductionism

# A First Characterization of Analogies

Analogies are frequently used in scientific descriptions and explanations.

Indicators for analogical reasoning and descriptions are comparing phrases:

- 'similar as'
- 'likewise'
- 'analogically'
- etc.

Let's begin with a short overview of the purposes of analogical reasoning and descriptions!

# The Purpose of Analogical Usage of Language

There are four main purposes of analogical usage of language (cf.: [Bun73], [Hem70], [Boc59] and [Wei76]):

F1 Abbreviation: e.g., in mathematics analogies are used for abbreviating proofs (cf. 'without limiting the generality').

F2 Didactics: illustration of claims. E.g.:

- Claims about a unknown domain $G_1$ (space-time-curvature by heavy masses)
- well-known domain $G_2$ (masses on elastic surfaces)

This mode of describing is very frequent in teaching and presenting scientific theories.

# The Purpose of Analogical Usage of Language

F3 Context of discovery: the value of analogies is to be found in the heuristics for finding new regularities; Hempel, e.g., claims:

> *"In order to appraise the explanatory significance of analogical models, and more generally of analogies based on nomic isomorphism, let us suppose that some "new" field of inquiry is being explored, and that we try to explain the phenomena encountered in it by analogical reference to some "old", previously explored domain of inquiry." (cf. [Hem70, p. 438])*

F4 Context of justification; E.g.:

- Someone wants to argue that a claim $A_1$ is a consequence of a theory $T_1$, but has no exact theoretical frame, proof etc. for this claim.
- Then she may show that there are very relevant relations of analogy between $T_1$ and another (well-known) theory $T_2$ and between $A_1$ and a consequence of $T_2$, call it $A_2$ ($T_2 \vdash A_2$).
- By establishing these analogies she then may claim: $T_1 \mid\!\sim A_1$.

# The Purpose of Analogical Usage of Language

F1–F3 suggest that analogical usage of language is principally (i.e. without consideration of psychological facts of restricted imagination power, hypotheses invention, demonstration power etc.) redundant in science.

Different investigations of F4 come to different results about the value of analogical usage of language: there are two opposing parties.

One group (of philosophers of science) does *not* accept analogies for the justification of theories. E.g.: Duhem in [Duh98] and Hempel in [Hem70, chpt.6].

Whereas another group accepts analogies for the justification of theories. E.g.: Bocheński in [Boc59].

A similar difference of opinions seems to hold also for concept formation by analogies (e.g.: is the meme-gene-analogy acceptable for speaking of meme theories?).

# A More Detailed Characterization of Analogies

Comparison of water in a pipe with current in a conductor:

Shortened analogical description:
'Electric current in a conductor is like water in a pipe.'

Take, e.g., the law of Hagen-Poiseulle and Ohm's law:

L1 $p_1 - p_2 = \frac{V}{c}$ ($V\ldots$ volume of fluid, $c\ldots$ speed, $p_i\ldots$ pressure)

L2 $v_1 - v_2 = \frac{I}{k}$ ($I\ldots$ amperage, $k\ldots$ conductance, $v_i\ldots$ potential)

# A More Detailed Characterization of Analogies

It is well known that $c$ varies indirect proportional with the length of the pipe:

L3 $c \sim \frac{1}{l_1}$ ($l_1 \ldots$ length of the pipe)

Analogical to this fact it holds that $k$ varies indirect proportional with the length of the conductor:

L4 $k \sim \frac{1}{l_2}$ ($l_2 \ldots$ length of the conductor)

Furthermore it holds that:

L5 $V \sim r_1^4$ ($r_1 \ldots$ radius of the pipe)

But it holds (not similarly) that:

L6 $I \sim r_2^2$ ($r_2 \ldots$ radius of the conductor)

# A More Detailed Characterization of Analogies

Analogical usage of language about two different domains (e.g., physics of liquids and electromagnetism) is given here in the sense that some descriptions of regularities are syntactically isomorph, that is:
$V \mapsto I$, $c \mapsto k$, $p_i \mapsto v_i$ and vice versa.
With the help of this example the main problem of analogical usage of language is easily expressed:

*Which descriptions of regularities within one domain of investigation are adequately adoptable for descriptions of regularities within another domain of investigation?*

The simplest solution to the problem would be a restrictive definition (cf. [Hem70, p.434]):

*Instead of defining 'x is analogue to y' one just defines 'x is analogue to y with respect to $L_i$'.*

According to this solution it holds: $V$ is analogue to $I$ with respect to L1 and L2, but not with respect to L5 and L6.

# A More Detailed Characterization of Analogies

Let *is* be a (partial) mapping (on the vocabulary of both theories):

- $is(I) = V$
- $is(v_i) = p_i$
- $is(k) = c$
- $is(l_2) = l_1$

Then one may generalize *is* inductively:

- For all ...: $is(P^n(t_1, \ldots, t_n)) = is(P^n)(is(t_1), \ldots, is(t_n))$
- For all terms $t_1, t_2$: $is(t_1 \equiv t_2) = is(t_1) \equiv is(t_2)$
- For all formulas $A$: $is(\neg A) = \neg is(A)$
- For all formulas $A, B$: $is(A \& B) = is(A) \& is(B)$
- For all formulas $A$ and variables $x$: $is(\forall x A) = \forall x is(A)$

And describe the analogical relations by: L1$\Rightarrow$ $is$(L1), L3$\Rightarrow$ $is$(L3)

# Concept Formation by Analogies

What does it mean that by these analogical relations current ($I$) and conductance $k$ are in some way characterized?

The analogical relations can be restated logically equivalent as:

- L1$\Rightarrow$ ($is$(L1) $\Leftrightarrow$L1)
- L3$\Rightarrow$ ($is$(L3) $\Leftrightarrow$L3)

Which may be seen as conditionalized contextual definitions of:
$I$, $k$, $v_i$ and $l_2$

Perhaps by such restatements one can make some sense of 'concept formation by analogies'.

Main problems:

- conditionalized multiple characterization of an expression
- difference between contextual definitions and non-definitional axioms

# REDUCTIONISM

# A Preliminary Distinction

Within the discussion of reductionism one should distinguish more or less sharply between the methods of (cf. [Mou08, p.79]):

- Elimination
- Reduction
- Definition

Some examples for a first characterization:

- Elimination of some theoretical terms by the method of Ramsey
  Frustration aggression theory (cf. [Dol+70]): $T = \{\forall x(Frus(x) \rightarrow Aggr(x)), \forall x(SOff(x) \rightarrow Frus(x)), \forall x(Aggr(x) \rightarrow (Shou(x) \vee Hitt(x) \vee \ldots))\}$
  $$\Downarrow$$
  $$T^R = \exists P \exists Q(\forall x(P(x) \rightarrow Q(x)) \wedge \forall x(SOff(x) \rightarrow P(x)) \wedge \forall x(Q(x) \rightarrow (Shou(x) \vee Hitt(x) \vee \ldots)))$$
  Result: $T^R$ without a specific theoretical vocabular and $empCont(T) = empCont(\{T^R\})$
- Reduction of Thermodynamics to Statistical Mechanics (cf. the discussion in [Nag79, chpt. 11])
- Definition of ‘is an ordered pair’ by the method of Wiener/Kuratowski

In the following we are going to talk about reductions and definitions, but not about elimination in a broad sense.

# Classical Reductionism

# Different Kinds of Reductionism

There are different positions subsumed under the label ‘reductionism’. One may categorize the most important positions in the following way (similar to [Cra00]):

- Ontological reduction:
  That is to identify all objects of the domain of one theory with some objects of the domain of another theory.
- Translational reduction:
  - R1 Term-by-term translation
  - R2 Sentence-by-sentence translation
  - R3 Law-by-law translation
  - R4 Theory-by-theory “translation” (mostly called ‘explanational reduction’)

Once again we make a restriction: we will only consider translational reduction and stick mainly to term-by-term translations.

NB: There are also more general distinctions of reductionism (cf. the introduction in [Cha92] and [Cat07]):
normative (ideal of science) vs. descriptive (real development of science)

# Different Kinds of Translational Reductions

Given this categorization, the main task to do is to clarify what is meant by 'translation'.

Classical answers to this problem are as follows:

R1.1 An expression $y$ is translatable to a set of expressions $X$ iff $y$ is definable with the help of $X$.

R1.2 An expression $y$ is translatable to a set of expressions $X$ iff $y$ is partially definable with the help of $X$.

R1.3 An expression $y$ is translatable to a set of expressions $X$ iff $y$ is connectable to elements of $X$ by so-called rules of correspondence.

$\vdots$

Relative to these clarifications there hold different relations between the different kinds of translational reduction.

E.g., in case of interpreting 'translation' in the sense of R1.1:
R1$\Rightarrow$R2, R2$\Rightarrow$R3, R2$\Rightarrow$R4, R3$\not\Rightarrow$R4, …

# NON-CLASSICAL REDUCTIONISM

# Further Problems for Classical Reductionism

The weaker the criteria for translations are, the more one looses important formal features of reductions (relative completeness, relative consistency etc.).

Beside such formal problems there are also posed some more informal ones.

E.g., one of the main objections against physicalism ($\phi \Rightarrow \psi$) are the following ones (cf. [Bec01, p.90]):

1. Mental predicates are cluster concepts— there are no sufficient and necessary conditions for defining them physicalistically.
2. If one tries to define them, then one produces a circle—at least in describing test-reaction-pairs.
3. Mental predicates are at the best only partially definable.

As far as 2 seems to be discussable only with respect to single cases, and as far as 3 seems to be addressed at least partly by reductions weaker than R1.1, we are going to concentrate only on 1.

# The Problem of Finding Adequate Conditions

The objection against classical reductionism in 1 is justified by the claim that—to give an example—$SOff(c_1)$ sometimes leads to $Shou(c_1)$ or $Hitt(c_1)$ or ..., but not always, and that because of this such reductions are inadequate (cf. [Bec01, pp.87f]).

We may demonstrate this objection by the given example of the R1.3-reduction of the frustration aggression theory:

In detail, the argument runs against the supposition about tests made within R1.3-reductions:

$\forall x(\exists t(SOff(x,t) \wedge (Shou(x,t) \vee Hitt(x,t) \vee \ldots)) \rightarrow \forall t(SOff(x,t) \rightarrow (Shou(x,t) \vee Hitt(x,t) \vee \ldots)))$

The most natural way to address this objection seems to try to overcome this problem by weakening this supposition about tests:

$\forall x(\exists t(SOff(x,t) \wedge (Shou(x,t) \vee Hitt(x,t) \vee \ldots)) \rightarrow$ *usually it holds for* $t(SOff(x,t) \rightarrow (Shou(x,t) \vee Hitt(x,t) \vee \ldots)))$, etc.

# Non-Classical Reductionism

Such a weakening corresponds to a weakening of the requirements for R1.3-reductions.

One may try, e.g.:

*Usually it holds for x and* $t(SOff(x,t) \rightarrow (Aggr(x) \leftrightarrow (Shou(x,t) \vee Hitt(x,t) \vee \ldots)))$

And this is to allow not only reductions within classical logic, but also within non-classical logic:

**Definition (Non-classical term-by-term reduction)**

An expression $t$ of $T_2$ is reducable to a set of expressions of $T_1$ iff $t$ of $T_2$ is non-classically connectable via so-called rules of correspondence with expressions of $T_1$.

# Extended Non-Classical Reductionism

# Different Kinds of R4-Reductions

Different kinds of theory-by-theory reductions (explanational reductions):

R4.1 The strictest forms of reductions are derivations: $T_1 \vdash T_2$

R4.2 A more moderate form of a reduction is definitional derivation. Let $D$ be a set of definitions of some concepts of $T_2$ with the help of concepts of $T_1$. Then reduction in this sense is a demonstration of $T_1 \cup D \vdash T_2$.

R4.3 A yet more moderate form of reductionism is definitional and reductional derivation. Let $D$ be a set as described above and $R$ be a set of reduction sentences interrelating some concepts of $T_2$ with concepts of $T_1$ (e.g., by meaning postulates, bilateral reduction sentences etc.). Then reduction in this sense is a demonstration of $T_1 \cup D \cup R \vdash T_2$.

$\vdots$

# An Extension of Non-Classical Reductionism

If the meaning of ‘concept formation by analogies’ is clarified in the indicated way, one may try to provide reductions by analogies.

(A research programme could be, e.g., the reduction of meme theories to gene theories by the help of analogies.)

R4.4 Let $D$ and $R$ be as described above and let $A$ be a set of concept formations of some concepts of $T_2$ by analogies with respect to the concepts of $T_1$. Then analogical reduction is a demonstration of $T_1 \cup D \cup R \cup A \vdash T_2$.

One may also try to find out which criteria of theory reduction/extension are satisfied by such reductions/extensions (of course not: eliminability etc.)

# Summary

- There are two ways of using analogies in science:
  - Conclusions by analogies
  - Concept formation by analogies
- Needed: An explication of 'concept fromation by analogies'. (wip)
- One possible solution: partial contextual definitions – further investigations about formal properties are needed. (wip)
- Relevant modern context for conclusions by analogies: Bayesianism.
- Relevant modern context for concept formation by analogies: Reductionism. (wip)
  - Possible applications: Supervenience thesis (the meme-gene-analogy is in support of the claim that cultural evolution supervenes biological evolution) etc. (wip)

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