Laying the Foundations for a Frame-Theoretic Notion of Reduction in Science

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The Aim, The Plan, The Proviso

- **The Aim**: The aim of this talk is to lay the foundations for a frame-theoretic notion of reduction in science. The *realistic* aim of this talk is to lay the foundations for a correct account of reduction in science.

- **The Plan**:

  Part I: The Classical Concept of Reduction
  Part II: Three Objections
  Part III: The Neo-Classical Concept and its Solutions
  Part IV: Liberalising the Classical Concept Further
  Part V: A Very Rough Sketch of Reduction in Frame-Theoretic Terms

- **The Proviso:** One qualification that I will make from the outset is that our focus in this talk will be on diachronic inter-theory reduction.

Part I:
The Classical Concept of Reduction

Derivability and Connectability

- The classical conception of reduction goes back to Nagel (1961). According to this conception, a theory T reduces to a theory T' if, and only if, two conditions are met:

  (i) connectability: for every term F in T, there is a term G that is constructible in T' such that for any object a, Fa if, and only if, Ga

  and

  (ii) derivability: T is derivable from T', potentially bridge laws B and potentially restrictive conditions A.

- Nagel identified two types of reduction that satisfy the above explication, namely **homogenous** and **heterogeneous** (a.k.a. 'inhomogenous') reductions.

Homogenous Reductions

- **Homogeneous** reductions are those where the reduced theory's vocabulary is either included in, or at least can be defined in terms of, the reducing theory's vocabulary.

  Example: The reduction of Galileo's law of free fall to Newtonian physics. Since the former assumes that acceleration is constant at or near the Earth's surface while the latter takes it to be proportional to the force acting on the given body, a restrictive condition is required for the derivation. This takes the form of the constant *g*, which denotes the 'average' acceleration imparted on objects with small mass by the Earth's local gravitational field.

Heterogenous Reductions

- **Heterogeneous** reductions require bridge laws to meet the connectability condition. Bridge laws connect the vocabulary of the reduced and reducing theories so that derivability can be achieved. In other words, bridge laws come into play only in heterogeneous reductions.

  Example: The reduction of the Boyle-Charles law to statistical mechanics. The required bridge law connects temperature (a concept in thermodynamics) with mean kinetic energy (a concept in statistical mechanics).

$$pV = \frac{2n}{3}\langle E_{\mathrm{kin}}\rangle \qquad T = \frac{2n}{3k}\langle E_{\mathrm{kin}}\rangle \qquad p\,V = k\,T$$

  NB: The derivations are often long to go through and involve a number of assumptions. The most important and controversial step is the introduction of a bridge law, as this is *not* included in the original resources of the reducing theory.

Part II:
Three Objections

Questioning Derivability

- A number of objections have been raised against the classical conception of reduction. We will be looking at three such objections. They all appear in Feyerabend (1962) but for more direct and indirect critiques see Kuhn (1962), Field (1973) among others.

- The first one concerns the derivability requirement. Feyerabend points out that in the great majority of cases in actual science this requirement cannot be met because the reduced theory and the reducing theory are inconsistent.

  <u>Example</u>: Strictly speaking, we cannot derive Galileo's law of free fall from Newtonian physics for, even when the restrictive condition of considering objects at or near the Earth's surface is taken into account, the values predicted by the two theories are different. In other words the two theories are inconsistent.

Questioning Semantic Invariance

- The second objection is related to the first and it concerns the variance of meaning across theories. Feyerabend argued that if meaning holism holds, then differences in the semantic content of theories imply differences in the semantic content of all their terms. This coupled with the view that the reference of a term is determined by its semantic content entailed that a term appearing in two theories cannot be referring to the same object. Thus the connectability condition cannot hold.

  Example: Although the concept mass appears in both classical mechanics and the special theory of relativity they do not mean exactly the same thing. In the latter case (relativistic) mass is not an invariant quantity but increases as the velocity of an object nears that of the velocity of light.

Questioning the Bridge Laws

- The third objection or concern is about the status of bridge laws. Are these supposed to be conventional stipulations, analytical truths or synthetic (i.e. empirical) truths and why? If synthetic, what is the warrant for endorsing them? Moreover, are they supposed to be identity claims, equating one class of objects with another, or is it enough that they merely correlate the two classes?

- Nagel (1961) appears to be unclear regarding what he considers to be correct answers to these questions. But surely it is clear that such laws cannot be mere conventional stipulations as that would trivialise the whole issue of reduction. Even so, this still leaves us with quite a few options.

Part III:
The Neo-Classical Concept
and its Solutions

The Neo-Classical Account

- Solutions to these and other objections are discussed in various places. In what follows we focus on Schaffner (1967; 1976) and Dizadji-Bahmani et al. (2010). The latter builds on the neo-classical account of reduction articulated Schaffner, thereby offering the most sophisticated reply to the above objections up to now.
- Schaffner's main innovation was to point out that what gets reduced is not the original theory *T* but a corrected version *T** that is strongly analogous to *T*. We get *T** from *T´* after applying the necessary restrictive conditions and bridge laws.
- Contra Feyerabend, it can thus be argued that derivability is maintained, even though what gets derived from *T´* is *T**, not *T*. And if derivability is maintained that means that semantic invariance is maintained (at least in so far as the bridge laws set up semantic equivalences) – as before only between *T´* and *T**.
- On the subject of bridge laws, Schaffner insists that such laws or 'reductive functions', as he calls them, must establish a functional relation between the terms of *T** and *T´* such that: (i) the entities to which they apply are the same and (ii) the predicates that these entities satisfy are the same. He takes bridge laws to be synthetic identity claims. Nagel (1974) also takes the synthetic stance.

The Neo-Classical Account: Strong Analogies

- Obviously, the success or failure of this solution hangs on the notion of strong analogy. Alas, Schaffner is cagey on this front. Here are two telling quotes:

  "This last point [about strong analogy] is perhaps the most programmatic, for not much work of any import has been done on the logic of analogy" (1967, p. 146). He then cites Hesse (1966) "for some interesting beginnings" on the topic.

  "These relations of approximate equality, close agreement, and strong analogy have yet to find formally precise characterizations, and to date represent informal aspects of a reduction. These elements in the reduction should not, however, be taken as implying that the relation between the reducing theory and reduced theory, in its corrected form, is vague or imprecise." (1976, p. 617).

- It is noteworthy that Nagel (1974) also backs the idea of 'good approximations' when the reduced theory cannot be directly derived from the reducing theory, bridge laws and restrictive conditions.

Extending the Neo-Classical Account

- Dizadji-Bahmani et al. make two crucial modifications to Schaffner's model calling the resulting model the 'Generalised Nagel-Schaffner model' (GNS):

  (1) It is not necessary that every term of T^* be connected to a term in T'.

  (2) It is not necessary that a term of T^* be connected to exactly one term in T'.

- The first one allows the modeling of partial reductions. No clear rationale is provided for it, though it is obvious they want to model cases where "we can deduce only some laws (or central statements)" of the reduced theory T^* (p. 399).

Extending the Neo-Classical Account (2)

- The second one allows the modeling of multiply realisable relations between the reduced and the reducing theories. The rationale provided for it is the following: "Reductions are desirable first and foremost for two other reasons: consistency and confirmation. That is, T_F [the reducing theory] and T_P [the reduced theory] have to be consistent, and evidence confirming T_F also has to confirm T_P and vice versa" (p. 405).

  NB: Actually what Dizadji-Bahmani et al. need to say here is that T and T^*, or as they denote them T_F and T_P^* stand in such a relation.

  Example: "There simply is no reason to think that, say, 'temperature' for gas being co-extensional with mean kinetic energy precludes it from being co-extensional with a completely different micro-property in other systems" (p. 406).

Extending the Neo-Classical Account: Bridge Laws

- Dizadji-Bahmani et al. go on to say: "Reductions that achieve nothing but consistency and confirmation are bona fide reductions. These aims, and this is the crucial point, can be achieved without bridge laws being identity statements. In fact, mere de facto correlations between properties are all that is required for the needs of reduction, and we can remain agnostic about the question of whether bridge laws express anything beyond mere correlation" (p. 405).

  NB: This view is in direct opposition to that taken by Sklar: "the place of correlatory laws is taken by empirically established identifications of two classes of entities. Light waves are not correlated with electromagnetic waves, for they are electromagnetic waves" (1967, p. 120).

Part IV:
Liberalising the Classical Concept Further

A Little Mistake

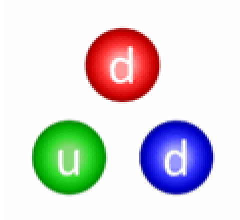
- We support the liberalisation of the notion of reduction carried out by Dizadji-Bahmani et al. Indeed we think that in order to capture all the subtleties involved in reductive relations the notion must be liberalised further.
- But before we embark on this project, it is important to point out a mistake in Dizadji-Bahmani et al. Their assertion that it is not necessary that every term of *T** be connected to a term in *T´* is strictly speaking false. That's because for there to be a derivation of *T** from *T´* (via bridge laws) all the terms of *T** must be connected to some terms of *T´*.

  "Schaffner's presentation of bridge laws suggests that he takes it to be the case that, in a successful reduction, (a) every term of T_P* is connected to a term of T_F, and that (b) a term of T_P* is connected to exactly one term of T_F (see, for instance, 1967, pp. 139–140). We take neither of these conditions to be necessary for a successful reduction" (p. 399).
- What they really must mean here is that it is not necessary that every term of *T*, the original theory, be connected to a term in *T´* (or even *T**). But this would be a moot point as, according to them, the bridge laws connect neither *T* and *T** nor *T* and *T´* but rather *T** and *T´*.

Not every *T´* term needs to be connected to *T**

- It is not necessary that every term of *T´* be connected to a term of *T** since *T´* may have additional terms in its vocabulary.

  Example: The concept of color charge in quantum chromodynamics has no connection to concepts in (corrected) classical physics. It is a property of quarks and gluons, elementary particles that were not even thought to exist within classical physics. Indeed, the property has nothing to do with color in vision and very little to do with classical conceptions of charge. Rather it is a property responsible for the strong nuclear interaction, an interaction that takes place at the very small level of quarks, and helps explain how the nucleus of an atom does not pull apart.



NB: This is a rather trivial and minor point since valid derivations need not involve the whole content of a reducing theory. And, of course, if only some content is derived then only some terms may be involved in the derivation.

Not every *T´* term needs to be connected to exactly one *T** term

- It is not necessary that a term of the reducing theory *T´* that is involved in the reduction to be *directly* connected to exactly one term of the corrected theory *T** or even *indirectly* connected (via analogy, not bridge laws) to exactly one term of the original theory *T*.

  Example: In relativistic physics the concept of energy is connected not only to the classical conception energy but also to that of rest (/inertial) mass via the relativistic mass-energy equivalence principle:

$$E = m_o \gamma c^2$$

  where E is the total energy of the system, m_o is its rest mass, γ the Lorentz factor and c the speed of light.

There may be Two or More Reducing Theories

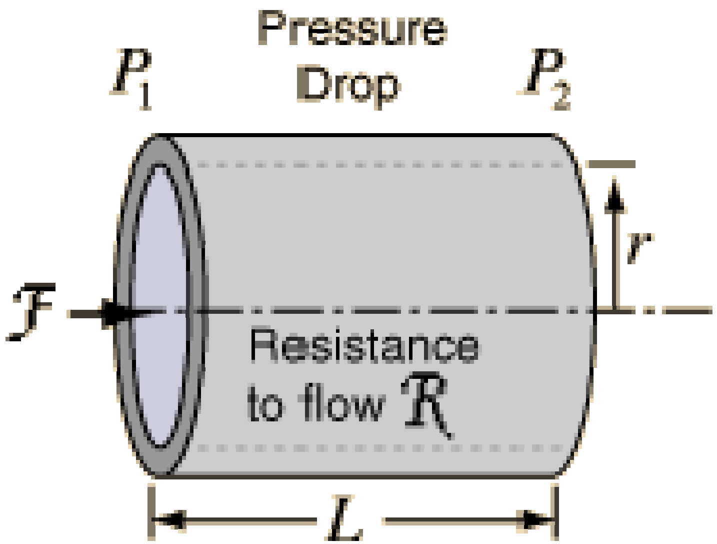
- Indeed it is not even necessary that a term of the corrected *T** / original theory *T* be directly / indirectly connected to terms in exactly one reducing theory *T´*.
- Typically successor theories unify existing domains of phenomena as well as new ones. For example, quantum physics can help explain the covalent bonding of molecules but it can also explain what happens in sub-nucleus interactions.
- At the same time, a successor theory need not unify all existing domains of phenomena. In the case at hand, quantum physics does not have anything to say about gravitational phenomena. That task falls to the general theory of relativity. Thus, it should not be expected that a term that appears in *T** or *T* to be directly / indirectly connected to exactly one reducing theory *T´*.

  Example: The concept of mass appears in both quantum and relativistic physics.

  NB: As already noted, there is something missing from quantum physics, namely a description of gravitational interactions. That's why physicists are currently working on quantum gravity. The hope is that a theory will emerge that unifies gravitational phenomena with phenomena relating to the other three forces.

Or None at All

- Some parts of old theories are genuinely successful but cannot be connected to any existing theory. Hans Radder (1996, p. 63) offers one such example in the guise of Poisseuille's law. The law cannot be derived from QM accounts of fluids. It seems to be a bona fide case of Kuhn loss – see Votsis (2011).



$$Q = \pi r^4 P / 8 \eta L$$

Poiseuille's law determines an (nearly) incompressible fluid's rate of laminar flow Q along a tube as a relation between the following quantities: the fluid's viscosity (measure of resistance) η, the radius r and length L of the tube and the pressure difference between the tube's two ends P.

Strongly Analogous Relations

- Despite their helpful modifications to the neo-classical model, Dizadji-Bahmani et al. leave us in the blind concerning the notion of strong analogy.

  "Being strongly analogous is a contextual relation, and we should not expect there to be a general theory of analogy... it is the particular science at stake that has to provide us with a criterion of relevant similarity in the particular context" (2010, p. 409).

- Notice that by making *T** satisfy the original conditions of reducibility, the burden of the whole reduction is put on the relationship between *T** and *T*. So, it is absolutely crucial to provide a crisp and defensible account of the notion of 'strongly analogous'.

- In other words, throwing light on the notion of 'strongly analogous' is a major sticking point in accepting the neo-classical model, at least as it is currently formulated.

Degrees of Analogousness

- Our proposal is to allow gradations in the analogousness of reductive relations. That is to say, the more non-trivial content continuity between *T** and *T*, the stronger the analogy between them.
- Why advocate a graded approach?
- First of all, note that *requiring* all the content of a theory *T* to be perfectly analogous to *T** clearly sets the bar too high. That's only the limit case.
- So, if we DO NOT want to turn analogousness into a trivially false affair, we must require less content continuity, i.e. we must lower the bar.
- Once the decision is made to lower the bar of what counts as an admissible level of analogousness , the question becomes where to draw the line.
- Ideally, we would want to find a principled way to do this. For example, we could require that at least some essential features of *T** are analogous to some essential features of *T*.

Avoiding Arbitrary Line-Drawing

- Proposals like this are ultimately indefensible. To see why this is the case, consider some (non-trivial) content of a theory which, as it so happens, represents its target system perfectly. Any deviation from this content – in terms of removing or changing but not adding content – means a less than perfect match between that content and the target system. So *any* such content (either essential or inessential) affects judgments of correspondence between the theory and the world. But the same holds for the correspondence between theories. Thus, no content should be ignored in assessing the level of reduction.

- **Punch-line**: Instead of drawing the line in an arbitrary way, it is better to embrace all forms of (non-trivial) continuity as equally legitimate albeit of different strengths.

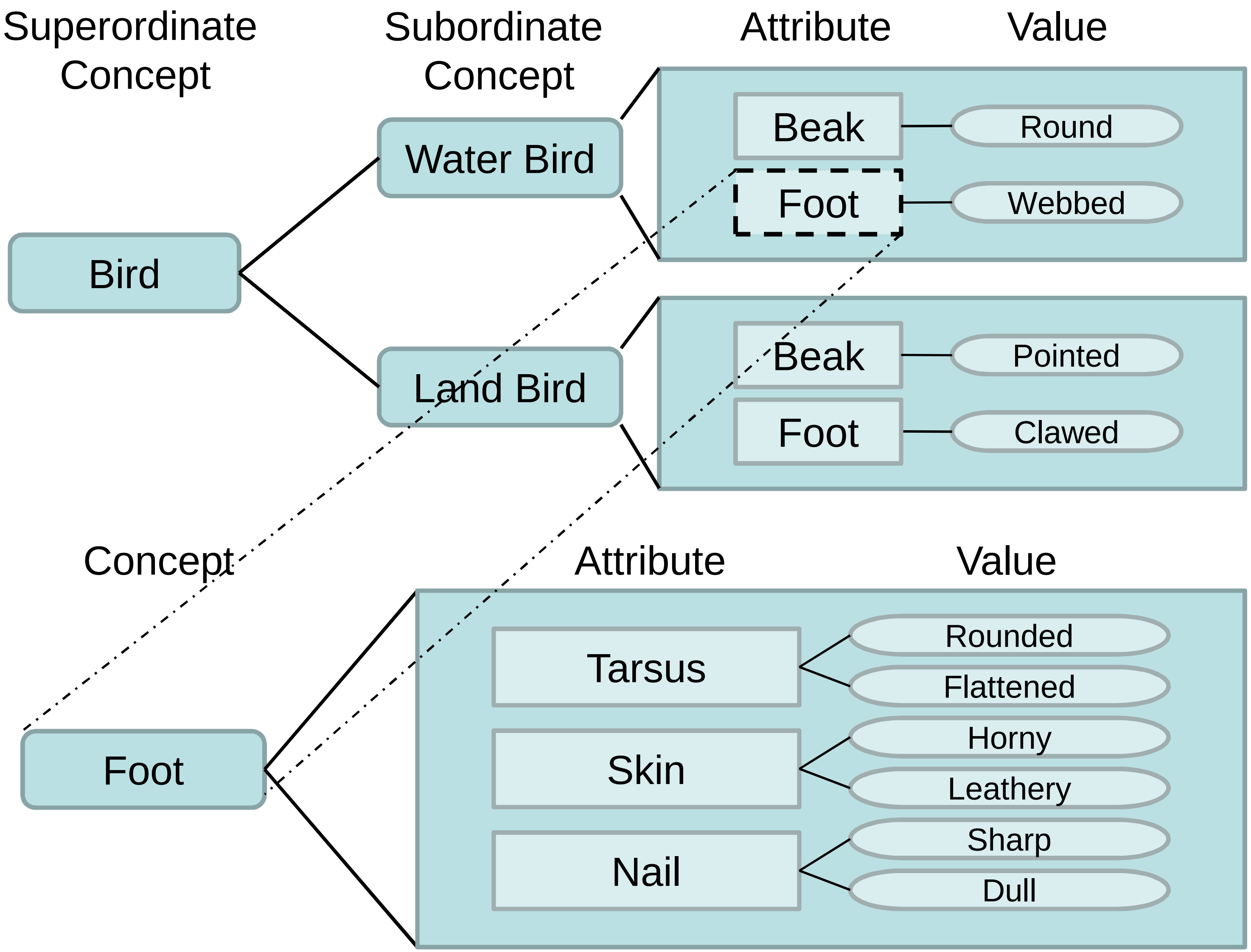
- **Broader Implications:** When claims about reduction are used as means to argue for or against scientific realism, progress in science, etc. then the presence or absence of strongly analogous relations is a more pressing matter. But even in those debates the strictness with which we must judge the strength of the analogousness is moderated by the fact that there need only be a general trend towards the cumulativity of scientific knowledge.

Part V:
A Very Rough Sketch of Reduction in Frame-Theoretic Terms

Frame Theory: The Basics

- A frame is a hierarchical structure that represents ordinary and scientific concepts by a system of attributes (Barsalou 1992).
- The nodes of a given frame may themselves be analysed into further frames. This feature makes frame theory a recursive system.
- In two recent papers, Votsis and Schurz (2011) and Schurz and Votsis (forthcoming) we employed the theory of frames to illustrate a certain amount of structural continuity between successive scientific theories.
- Consider the following frame for the concept BIRD taken from Andersen, Barker and Chen (2006).

Frame Theory: The Basics (2)



Why Frame Theory?

- One of frame theory's strengths is its ability to lay bare the inner structure of scientific concepts. This facilitates the task of comparing scientific theories because one can examine with relative ease whether frame-theoretically explicated concepts, their attributes and their values share structure. Such comparisons can reveal to what extent, if at all, two or more concepts are continuous and whether these concepts are incompatible and even radically incommensurable.

- As philosophers of science we find this ability very useful because one of the central aims of our discipline is to discover how scientific concepts of successive theories (and their respective ontologies) are related.

- As participants in the debate on reduction – and other cognate debates, e.g. the scientific realism debate – we are particularly interested to find out whether the relations (or the lack of relations) between the scientific concepts of successive theories uphold a strong reductive trend in science.

Reduction as Mappings Between Frames

- Our plan is to express the (suitably modified) neo-classical model of reduction in frame-theoretic terms. If each theory or theory-part involved in a reductive claim is expressible in terms of a frame hierarchy, we want to model the reductive relations between such hierarchies.
- As already noted, there are two steps in establishing a reductive relation between two theories in the neo-classical tradition. First, one must derive a corrected version *T** from *T´*. Second, one must show that there is an analogy of a certain strength between *T** and *T*. How are we to model these steps in frame-theory?
- A first (rough) approximation is that they can be modeled as follows:

<u>Step 1</u>: There is an isomorphic mapping between part of the structure of the frame hierarchy of *T´* (after the bridge laws are applied) and the whole structure of the frame hierarchy of *T**.

<u>Step 2</u>: There is a transformation function from the frame hierarchy of *T* to that of *T** that keeps some of the (non-trivial) content of the former intact. The more that remains invariant under the transformation function the stronger the analogy between the frame hierarchies of *T* and *T**.

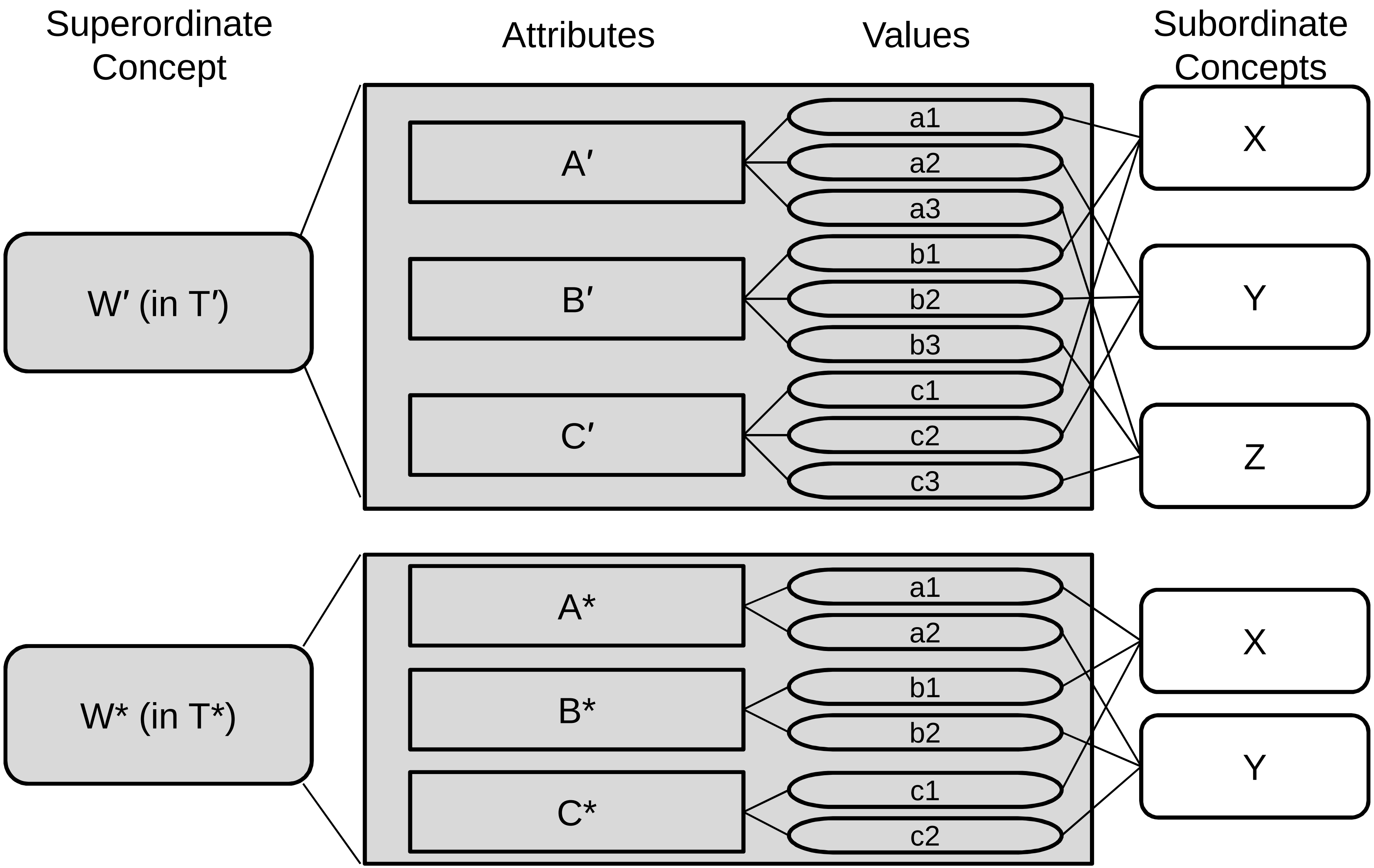
The Mapping Between Frame Hierarchies *T´* and *T**

- For the specific derivability relation to hold between *T´* (plus the bridge laws) and *T**, the content of *T** must be included in the content of *T´* (plus the bridge laws). At the same time, the content of *T´* must exceed that of *T** otherwise we would not need to talk about *T**, i.e. we would instead talk directly about the relationship between *T´* and *T*.

- We can express the content inclusion claim via an isomorphism between part of the frame hierarchy of *T´* (after the bridge laws are taken into account) and the whole of the frame hierarchy of *T** such that the values of any mapped attributes are not altered (but some values can be removed). Alternatively, we can talk about the embedding of *T** into *T´* (after the bridge laws are accounted for).

- In frame-theoretic terms this means that when compared to the frame hierarchy of *T´*, the frame hierarchy of *T** will possess less of (one or more of) the following: frame levels, concepts, attributes, values and the corresponding constraints (e.g. value-attribute, attribute-attribute and value-value constraints).

A Simple Example: Embedding *T** into *T´*



The Mapping Between Frame Hierarchies *T* and *T**

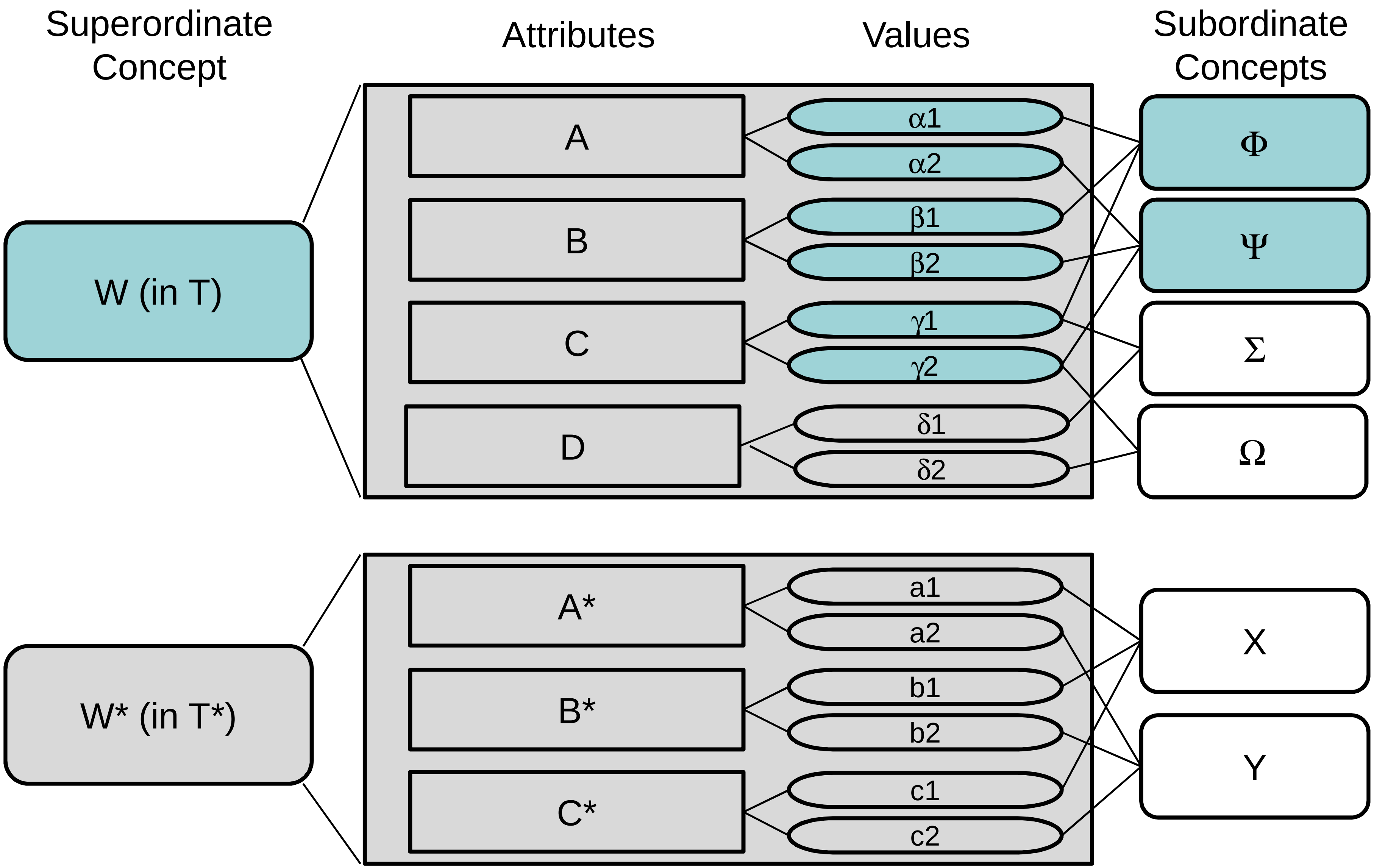
- The relation between *T* and *T** is more complicated. Here the relation we want to express is that of various degrees of analogousness.

- It was pointed out already that this can be achieved via a transformation rule from the frame hierarchy of *T* to the frame hierarchy of *T** that keeps some of the (non-trivial) content of the former intact. In general, the more that remains invariant under the transformation rule the stronger the analogy between the two frame hierarchies.

  NB: Ideally we want to make exact claims about the strength of the analogousness between frame hierarchies. To do that a metric is required. For any proposed metric to be adequate, it will need to yield judgments that do not change if we translate each frame-hierarchy into a notational variant.

- In frame-theoretic terms this means that the two frame hierarchies of *T* and of *T** will be continuous in at least some non-trivial respects. Even when none of the values of the mapped attributes are preserved, there may be a fairly strong analogousness in that the values are approximately the same. But of course they need not be since analogousness can come in various strengths.

A Simple Example: Transforming *T* into *T**



Thank you for Listening!